Geometry Unit 2 Reasoning and Proof
2-1 Conditional Statements

Conditional Statement – a statement which has a hypothesis and conclusion, often called an if-then statement. Conditional statements are contain two parts

Hypothesis – the part of the statement following " *if* "

Conclusion – the part of the statement following " *then* " (if "then" is present)

The following statements are conditional: Underline each hypothesis and circle each conclusion.

If the sun is shining, then I can go outside.

If an earthquake occurs, then I should hide under a table.

If I do my homework, then I might get a better grade.

I can go to the movies if I do my chores.

You live in California if you live in San Diego.

Writing a statement as conditional requires you to understand the nature of what you are talking about.

Take an accepted fact and translate it into a conditional statement.

Accepted Fact

A triangle has three sides.

Conditional Statement

If a figure is a triangle, then it has three sides.

Accepted Fact

An integer that ends in 0 is divisible by 10.

Conditional Statement

If an integer ends in a zero, then it is divisible by 10.

Accepted Fact

A regular pentagon has five congruent sides.

Conditional Statement

>____

Accepted Fact

A tiger is an animal.

Conditional Statement

>____

Conditional statements have a truth value.

Truth Value – the determination of whether a statement is true or false.

Counterexample - A specific example which proves a statement is false.

Not all conditional statements are true.

Example:

If it is February, then there are only twenty eight days in the month.

The statement is false.

Counterexample: February of 2012 had 29 days, it was a leap year.

Determine the truth value of the following statements. If false, provide a counterexample.

If an angle measures 37.25 · , then it is an obtuse angle.__________________________________

If x – 3 = 7, then x = 10.

____

If a point is in the first quadrant, then the coordinates are negative.

____

If the probability of an event occurring is 1, then it is certain to occur.

____

If two angles have the same measure, then they are supplementary.

____

If you live in a country bordering the US, then you live in Mexico.

____

If an integer is odd and less than 10, then it is a prime number.

____

Converse of a conditional statement: A statement which switches the hypothesis and conclusion of a conditional statement.

Note: The converse of a conditional statement may not have the same truth value.

Example: **Conditional Statement**

If two lines intersect to form right angles, then the two lines are perpendicular.

Converse

If two lines are perpendicular, then they intersect to form right angles.

**Both the original statement and converse are true in this case.

Conditional Statement: If today is Sunday, then it is the weekend.

Converse: If it is the weekend, then it is Sunday.

*The original statement was true, but the converse is false.

The counterexample would be “It could be Saturday.”

Inverse: When you negate (negative statement) the hypothesis and conclusion of a conditional statement.

If today is not Sunday, then it is not the weekend.

Contrapositive: When you negate the hypothesis and conclusion of the converse of a conditional statement.

If it is not the weekend, then it is not Sunday.

Try This: Write the converse, inverse and contrapositive to the statement:

Conditional Statement: If two lines do not intersect and are not parallel, then they are skew.

Converse:

Inverse:

Contrapositive:

2-2 Biconditionals and Definitions

Biconditional Statement – When a conditional statement and its converse are true, the statement can be written as a biconditional statement.

Example:

Conditional

If two angles have the same measure, then they are congruent.

Converse

If two angles are congruent, then they have the same measure.

Biconditional

Two angles have the same measure **if and only if** the angles are congruent.

Your turn:

Conditional: If three points lie on the same line, then they are collinear. True or false?

Converse:

____, True or False?

Biconditional, if conditional and converse are true.

____

Symbolic form

Type of statement	Symbollically written	Verbally read
Conditional	***p · q***	***If p, then q.***
Converse	***q · p***	***If q, then p.***
Inverse	***~p · ~q***	***If not p, then not q.***
Contrapositive	***~q · ~ p***	***If not q, then not p.***
Biconditional	***p if and only if q***	***p if and only if q.***

Good Definitions – statements that can aide you in clearly designating an object.

They include the following parts:

1) Uses clearly understood/defined terms (New ideas/terms are not presented here)

2) Is precise (Does not use general terms in the definition)

3) Is reversible. (Can be written as a biconditional)

The following is a good definition

A square is a figure with four congruent sides and four right angles.

1) It is defined in terms we have used previously.

2) It is precise and does not use any ambiguous terms.

3) It is reversible and can be written as a biconditional.

Conditional: If a figure is a square, then it has four congruent sides and right angles. **TRUE**

Converse: If a figure has four congruent sides and four right angles, then it is a square. **TRUE**

Biconditional: A figure is a square if and only if it has four congruent sides and four right angles.

Which of the three components of a good definition do the examples below fail?

Example: An airplane is a vehicle that flies.

Cannot be reversed, the converse is not true. Helicopters are vehicles that fly.

Try these:

A triangle has sharp corners. ____

A square is a figure with four right angles. ____

The asymptotes of a hyperbolic function are written in slope-intercept form. ________________________

2-3 Deductive Reasoning

Deductive Reasoning – the process of logically reasoning from given statements to form a conclusion.

Given statement – a statement which is provided for you as a starting point in a proof or discussion.

Example of deductive reasoning

A car is brought to a mechanic because it will not start. The mechanic has observed that when a car won't start it can often be a dead battery. The mechanic chooses to test the battery before checking any other components and find that the battery won't hold a charge.

What conclusion should be made here?

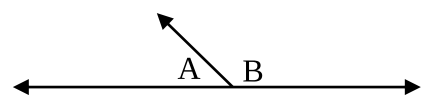
____

Should this be the end of the mechanics deductive reasoning? Why or why not?

____

Linear Pair Postulate – a pair of adjacent angles whose non-adjacent sides form a line.

$\angle A + \angle B = 180$



An example of deductive reasoning.

Given two angles form a linear pair. Those two angles are then forming a straight angle which measures 180 degrees. I know that supplementary angles are any two angles whose sum is 180 degrees.

What conclusion can be made?

____

Law of detachment - If a conditional is true, and its hypothesis by itself is true, then its conclusion must be true.

Symbolically: If *p* · *q* is true and *p* is true, then *q* is true.

Given: If there is the midpoint of a segment, then it divides it into 2 congruent segments.

Statement: J is the midpoint of line *m*. This proves *p* is true.

Conclusion: J divides line *m* into two congruent segments. Conclude that *q* is true.

Using the law of detachment, analyze the given and the statement, can a conclusion be made?

1) Given: If it rains, then practice will be canceled that day.

Statement: It rains on Sunday.

Can a conclusion be made? If yes, what is it? If no, why?____

2) Given: If a pitcher pitches a complete game, then his coach will not let him pitch the next day.

Statement: Tim Lincecum pitched a complete game on Sunday.

Can a conclusion be made? If yes, what is it? If no, why?____

Law of Syllogism – allows you to create a conditional statement from two true conditional statements when the conclusion of one statement is the hypothesis of the other statement.

Symbolic form: If $p \cdot q$, and $q \cdot r$ are true, then $p \cdot r$ is true

If I receive an A on my exam, then I will be able to go out this weekend.

If I am able to go out this weekend, then I will go to the football game.

Conclusion: If I get an A on my exam, then I will go to the football game.

Draw a conclusion to these statements using the law of Syllogism.

a) If two planes are not parallel, then they intersect.

If two planes intersect, then they intersect in a line.

Conclusion: ____

b) If you are studying biology, then you are studying a science.

If you are studying botany, then you are studying biology.

Conclusion: ____

2-4 Reasoning with Algebra

Properties of Equality: We assume that all the properties learned in Algebra 1 are true and factual.

Addition Property:	**If $a = b$, then $a + c = b + c$.**
Subtraction Property:	**If $a = b$, then $a - c = b - c$.**
Multiplication property:	**If $a = b$, then $ac = bc$.**
Division Property:	**If $a = b$, then a ÷ c = b ÷ c**
Reflexive property:	**$a = a$ (all numbers and variables are equal to themselves)**
Symmetric Property:	**If $a = b$, then $b = a$.**
Transitive Property:	**If $a = b$ and $b = c$, then $a = c$.**
Substitution Property:	**If $a = b$, then b can replace a in any expression or equation.**
Distributive property:	**If a(b + c), then ab + ac.**

Two – Column Proof – a proof with two distinct columns; the first column is a set of factual statements and the second column is the reasoning that allows the use of each factual statement.

Given: Two lines are Perpendicular.

Prove: Lines intersect at a 90 degree angle.

Statements	Reasons
Two lines are perpendicular	Given
They intersect at a right angle	Definition of Perpendicular Lines
All right angles measure 90 degrees	Definition of a Right Angle
The lines intersect at a 90 degree angles	Right Angle Congruence Theorem

Paragraph Proof - a proof which is written in a verbally descriptive way and applies logical deduction with the statements and reasons in a paragraph form.

Paragraph proof of the Congruent and Supplementary theorem:

Given: ∠A and ∠B are supplementary and congruent

Prove: ∠A and ∠B are right angles

Given ∠A and ∠B are supplementary, they would add to 180 · . Given the angles are congruent, then they must be equal in value according to the definition of congruent angles. That would give m∠A + m∠B = 180 · and m∠A = m∠B. m∠A could then be substituted for m · B according to the substitution property and you would have m∠A + m∠A = 180 · . Simplifying would be 2(m∠A) = 180 · . Using the division property of equality we would have get m∠A = 90 · . Given m∠A = m∠B, they would both be equal to 90 · . By definition of right angles both angles are right angles.

Algebraic Proofs

Solving a simple equation looks much different when the properties are labeled.

A Two-column proof is now applied with factual statements on the left and reasons on the right.

Given: $3(2x + 9) = 4x + 63$

Prove: $x = 18$

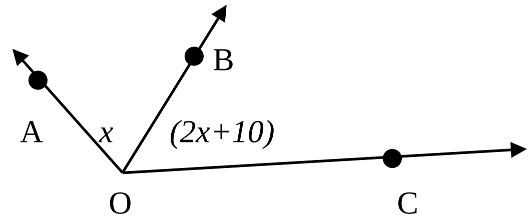
Statements	Reasons
$3(2x + 9) = 4x + 63$	Given
$6x + 27 = 4x + 63$	Distributive Property od Equality
$2x + 27 = 63$	Subtraction Property of Equality
$2x = 36$	Subtraction Property of Equality
$x = 18$	Division property of equality

Justifying steps in solving an equation.

Given: $m\angle AOC = 139$ ·

**Anything in the figure is also given

Prove: $x = 43$

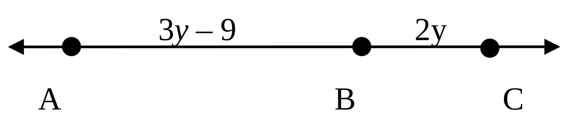


Statements	Reasons
$m\angle AOC = 139$	Given
$m\angle AOB + m\angle BOC = m\angle AOC$	Angle Addition Postulate
x · $+(2x + 10)$ · $= 139$ ·	Substitution Property of Equality
$3x + 10 = 139$	Simplify (or combine like terms)
$3x = 129$	Subtraction Property of Equality
$x = 43$	Division property of equality

Given: $AC = 19$

Prove: $y = 5.6$

**Anything in the figure is also given.



	Statements	Reasons
1)	______________	______________________
2)	______________	Segment Addition Postulate
3)	$3y - 9 + 2y = 19$	______________________
4)	______________	Simplify (or combine like terms)
5)	$5y = 28$	______________________
6)	$y = 5.6$	Division property of equality

2-5 Proving Segments Congruent

Proof – a group of statements and reasons which uses givens and a set of theorems to show a pre-determined conclusion.

Geometric Properties of *Congruence*

Reflexive Property	**a ≅ a**
Symmetric Property	**If a ≅ b , then b ≅ a**
Transitive Property	**If *a* ≅ *b* and *b* ≅ *c*, then *a* ≅ *c*.**

Try These: Answer the following questions using the properties of Algebra or Congruence

Addition Property of Equality
If $2x - 5 = 10$, then $2x =$ _?_.

Symmetric Property of Equality
If $AB = YU$, then _?_.

Reflexive Property of Congruence
$\angle PQR \cong$ _?_

Substitution Property
If $LM = 7$ and $EF + LM = NP$, then _?_ $= NP$.

Subtraction Property of Equality
If $5x + 6 = 21$, then _?_ $= 15$.

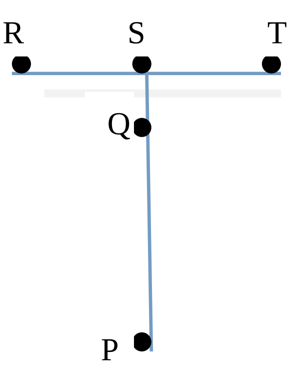
Symmetric Property of Congruence
If $\angle H \cong \angle K$, then _?_ $\cong \angle H$.

Distributive Property
$3(x - 1) = 3x -$ _?_

Transitive Property of Congruence
If $\angle XYZ \cong \angle AOB$ and $\angle AOB \cong \angle WYT$, then _?_.

Given: PQ ≅ RS, QS ≅ ST

Prove: PS ≅ RT



Statements	Reasons
1) ________________	1)Given
2) PQ = RS, QS = ST	2) ________________
3) ________________	3) ________________
4) ________________	4) ________________
5) PS = RT	5) Substitution Property of Equality
6) ________________	6) Definition of congruent segments

2-6 Proving Angles Congruent

Proof – a group of statements and reasons which uses givens and a set of theorems to show a pre-determined conclusion.

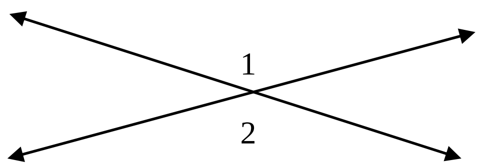
Theorem - A conjecture that is proven.

Corollary - A statement that follows directly from a theorem or definition.

Developmental Proof - A proof that shows why a theorem works. You cannot use that particular theorem in it's own proof.

Vertical Angles Theorem - Vertical angles are congruent.

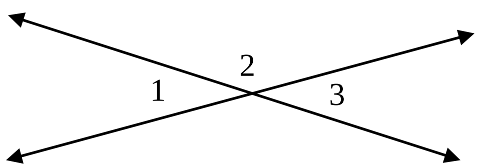
If $\angle 1$ and $\angle 2$ are vertical, then $\angle 1 \cong \angle 2$.



Developmental Proof of the Vertical Angles Theorem:

Given: $\angle 1$ and $\angle 3$ are vertical

Prove: $\angle 1 \cong \angle 3$



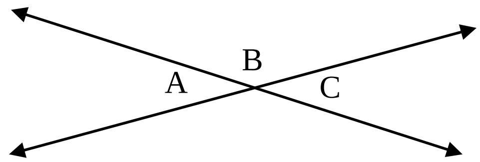
	Statements	Reasons
1)	$\angle 1$ and $\angle 3$ are vertical	1) Given
2)	$m\angle 1 + m\angle 2 = 180$	2) Linear Pair postulate
3)	$m\angle 2 + m\angle 3 = 180$	3) ________________________________
4)	$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	4) ________________________________
5)	$m\angle 1 = m\angle 3$	5) Subtraction property of equality
6)	$\angle 1 \cong \angle 3$	6) Definition of congruence statement

Congruent Supplements Theorem - If two angles are supplementary with the same angle, then the two angles are congruent.

Congruent Complement Theorem - If two angles are complementary with the same angle, then the two angles are congruent.

Right Angle Congruence Theorem - all right angles are congruent

Congruent and Supplementary Theorem – if two angles are congruent and supplementary, then they are both right angles.



Developmental Proof of the Congruent Supplements Theorem:

Given: ∠A and ∠B are supplementary, and ∠B and ∠C are supplementary

Prove: ∠A ≅ ∠C

	Statements	Reasons
1)	∠A and ∠B , ∠B and ∠C are supplementary	Given
2)	m∠A + m∠B = 180	Definition of supplementary angles
3)	m∠B + m∠C = 180	______________________
4)	m∠A + m∠B = m∠B + m∠C	______________________
5)	m∠A = m∠C	______________________
6)	__________	______________________

Developmental Proof of the Congruent Complements Theorem:

Given: ∠A and ∠B are complementary, and ∠B and ∠C are complementary

Prove: ∠A ≅ ∠C

	Statements	Reasons
1)	∠A and ∠B , ∠B and ∠C are Complementary	______________________
2)	m∠A + m∠B = 90	______________________
3)	______________________	______________________
4)	m∠A + m∠B = m∠B + m∠C	Transitive prop. of =
5)	__________	______________________
6)	__________	______________________

2-7 Indirect Proofs

Indirect Reasoning – Is a type of reasoning in which all possibilities are considered and then the unwanted ones are proved false. The remaining possibilities must be true.

Indirect Proofs - A statement and its negation often are the only possibilities.

A contradiction

We know that no mathematical statement can be both true and false at the same time.

Example: Let $x < 7$ and $x = 7$ such a statement would be a contradiction.

A negation

A negation of a statement says the opposite of the original statement.

Example: Statement (P): Let $x = 5$.

Negation (~P): Let $x \neq 5$

Indirect Proof

Step 1: Write the negation of the conclusion.

Step 2: Reason until you reach a contradiction and then explicitly state the contradiction.

Step 3: State that the assumption in Step 1 must have been false, therefore the original conclusion must be true.

1. Given: $2x - 3 > 7$

Prove: $x > 5$

Indirect Proof:

Step 1:

Assume that $x \leq 5$. That is, assume that $x < 5$ or $x = 5$

Step 2:

Make a table with several possibilities for x given that $x < 5$ or $x = 5$. This is a contradiction because when $x < 5$ or $x = 5$, $2x - 3 \leq 7$.

Step 3:

Therefore the assumption that $x \leq 5$ must be false, which means $x > 5$ must be true.

2. Given: $\angle A \neq \angle B$

Prove: $\angle A$ and $\angle B$ are not vertical angles.

Indirect Proof:

Step 1:

Assume that $\angle A$ and $\angle B$ are ________________

Step 2:

Then $\angle A$ ______ $\angle B$.

But the given says that $\angle A \neq \angle B$,

$\angle A$ ______ $\angle B$ and $\angle A \neq \angle B$ is a contradiction.

Step 3:

Therefore the assumption that $\angle A$ and $\angle B$ are vertical angles is false, and it must be true that ____.

3. Given: n is odd

Prove: n^2 is odd