# 7 Rational Functions



Cost of Fuel *(p. 397)*

Galapagos Penguin *(p. 382)*

Lightning Strike *(p. 371)*


3-D Printer *(p. 369)*

Volunteer Project *(p. 362)*

# Maintaining Mathematical Proficiency

## Adding and Subtracting Rational Numbers

**Example 1** **Find the sum $-\frac{3}{4} + \frac{1}{3}$.**

$$-\frac{3}{4} + \frac{1}{3} = -\frac{9}{12} + \frac{4}{12}$$ Rewrite using the LCD (least common denominator).

$$= \frac{-9 + 4}{12}$$ Write the sum of the numerators over the common denominator.

$$= -\frac{5}{12}$$ Add.

**Example 2** **Find the difference $\frac{7}{8} - \left(-\frac{5}{8}\right)$.**

$$\frac{7}{8} - \left(-\frac{5}{8}\right) = \frac{7}{8} + \frac{5}{8}$$ Add the opposite of $-\frac{5}{8}$.

$$= \frac{7 + 5}{8}$$ Write the sum of the numerators over the common denominator.

$$= \frac{12}{8}$$ Add.

$$= \frac{3}{2}, \text{ or } 1\frac{1}{2}$$ Simplify.

### Evaluate.

1. $\frac{3}{5} + \frac{2}{3}$
2. $-\frac{4}{7} + \frac{1}{6}$
3. $\frac{7}{9} - \frac{4}{9}$
4. $\frac{5}{12} - \left(-\frac{1}{2}\right)$
5. $\frac{2}{7} + \frac{1}{7} - \frac{6}{7}$
6. $\frac{3}{10} - \frac{3}{4} + \frac{2}{5}$

## Simplifying Complex Fractions

**Example 3** **Simplify $\frac{\frac{1}{2}}{\frac{4}{5}}$.**

$$\frac{\frac{1}{2}}{\frac{4}{5}} = \frac{1}{2} \div \frac{4}{5}$$ Rewrite the quotient.

$$= \frac{1}{2} \cdot \frac{5}{4}$$ Multiply by the reciprocal of $\frac{4}{5}$.

$$= \frac{1 \cdot 5}{2 \cdot 4}$$ Multiply the numerators and denominators.

$$= \frac{5}{8}$$ Simplify.

### Simplify.

7. $\frac{\frac{3}{8}}{\frac{5}{6}}$
8. $\frac{\frac{1}{4}}{-\frac{5}{7}}$
9. $\frac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{4}}$

10. **ABSTRACT REASONING** For what value of $x$ is the expression $\frac{1}{x}$ undefined? Explain your reasoning.

## Mathematical Practices

*Mathematically proficient students are careful about specifying units of measure and clarifying the relationship between quantities in a problem.*

## Specifying Units of Measure

### Core Concept

**Converting Units of Measure**

To convert from one unit of measure to another unit of measure, you can begin by writing the new units. Then multiply the old units by the appropriate conversion factors. For example, you can convert 60 miles per hour to feet per second as follows.

old units → $\dfrac{60\text{ mi}}{1\text{ h}} = \dfrac{?\text{ ft}}{1\text{ sec}}$ ← new units

$$\frac{60\cancel{\text{ mi}}}{1\cancel{\text{ h}}} \cdot \frac{1\cancel{\text{ h}}}{60\cancel{\text{ min}}} \cdot \frac{1\cancel{\text{ min}}}{60\text{ sec}} \cdot \frac{5280\text{ ft}}{1\cancel{\text{ mi}}} = \frac{5280\text{ ft}}{60\text{ sec}}$$

$$= \frac{88\text{ ft}}{1\text{ sec}}$$

### EXAMPLE 1 Converting Units of Measure

You are given two job offers. Which has the greater annual income?

- \$45,000 per year
- \$22 per hour

**SOLUTION**

One way to answer this question is to convert \$22 per hour to dollars per year and then compare the two annual salaries. Assume there are 40 hours in a work week.

$$\frac{22\text{ dollars}}{1\text{ h}} = \frac{?\text{ dollars}}{1\text{ yr}}$$ Write new units.

$$\frac{22\text{ dollars}}{1\cancel{\text{ h}}} \cdot \frac{40\cancel{\text{ h}}}{1\cancel{\text{ week}}} \cdot \frac{52\cancel{\text{ weeks}}}{1\text{ yr}} = \frac{45{,}760\text{ dollars}}{1\text{ yr}}$$ Multiply by conversion factors.

▶ The second offer has the greater annual salary.

## Monitoring Progress

1. You drive a car at a speed of 60 miles per hour. What is the speed in meters per second?
2. A hose carries a pressure of 200 pounds per square inch. What is the pressure in kilograms per square centimeter?
3. A concrete truck pours concrete at the rate of 1 cubic yard per minute. What is the rate in cubic feet per hour?
4. Water in a pipe flows at a rate of 10 gallons per minute. What is the rate in liters per second?

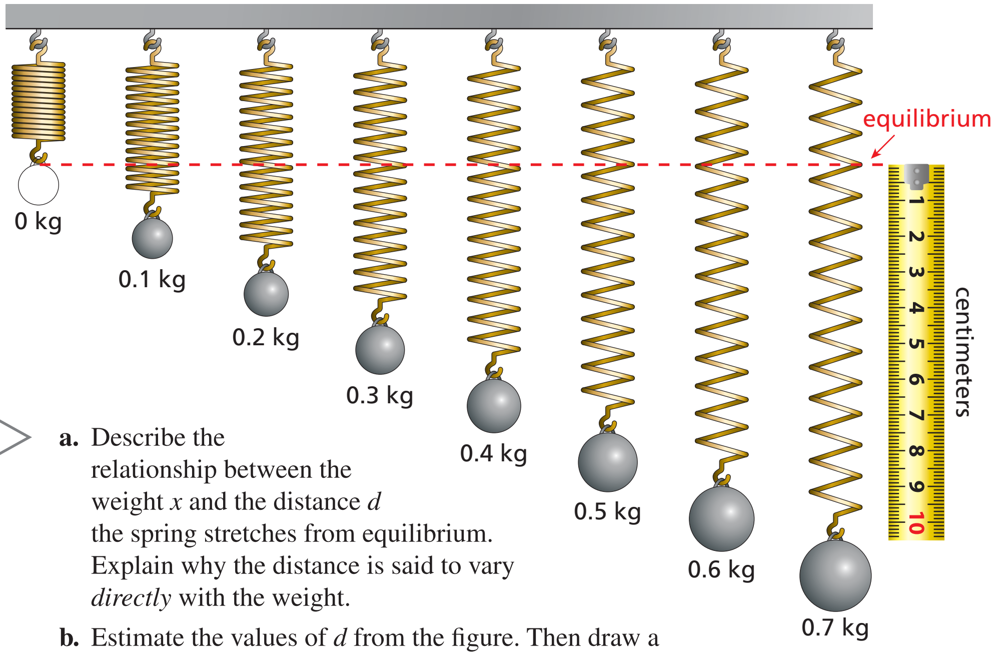
# 7.1 Inverse Variation

## Essential Question

How can you recognize when two quantities vary directly or inversely?

### EXPLORATION 1 Recognizing Direct Variation

**Work with a partner.** You hang different weights from the same spring.




**REASONING QUANTITATIVELY**

To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

**a.** Describe the relationship between the weight $x$ and the distance $d$ the spring stretches from equilibrium. Explain why the distance is said to vary *directly* with the weight.

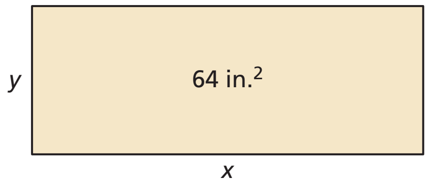
**b.** Estimate the values of $d$ from the figure. Then draw a scatter plot of the data. What are the characteristics of the graph?

**c.** Write an equation that represents $d$ as a function of $x$.

**d.** In physics, the relationship between $d$ and $x$ is described by *Hooke's Law*. How would you describe Hooke's Law?

### EXPLORATION 2 Recognizing Inverse Variation

**Work with a partner.** The table shows the length $x$ (in inches) and the width $y$ (in inches) of a rectangle. The area of each rectangle is 64 square inches.




| $x$ | $y$ |
|---|---|
| 1 | |
| 2 | |
| 4 | |
| 8 | |
| 16 | |
| 32 | |
| 64 | |

**a.** Copy and complete the table.

**b.** Describe the relationship between $x$ and $y$. Explain why $y$ is said to vary *inversely* with $x$.

**c.** Draw a scatter plot of the data. What are the characteristics of the graph?

**d.** Write an equation that represents $y$ as a function of $x$.

## Communicate Your Answer

**3.** How can you recognize when two quantities vary directly or inversely?

**4.** Does the flapping rate of the wings of a bird vary directly or inversely with the length of its wings? Explain your reasoning.

# 7.1 Lesson

## Core Vocabulary

inverse variation, *p. 360*
constant of variation, *p. 360*

***Previous***
direct variation
ratios

## What You Will Learn

- Classify direct and inverse variation.
- Write inverse variation equations.

## Classifying Direct and Inverse Variation

You have learned that two variables $x$ and $y$ show direct variation when $y = ax$ for some nonzero constant $a$. Another type of variation is called *inverse variation*.

### Core Concept

**Inverse Variation**

Two variables $x$ and $y$ show **inverse variation** when they are related as follows:

$$y = \frac{a}{x},\ a \neq 0$$

The constant $a$ is the **constant of variation**, and $y$ is said to *vary inversely* with $x$.

### EXAMPLE 1 Classifying Equations

Tell whether $x$ and $y$ show *direct variation*, *inverse variation*, or *neither*.

**a.** $xy = 5$

**b.** $y = x - 4$

**c.** $\frac{y}{2} = x$

**STUDY TIP**

The equation in part (b) does not show direct variation because $y = x - 4$ is not of the form $y = ax$.

**SOLUTION**

| Given Equation | Solved for $y$ | Type of Variation |
|---|---|---|
| **a.** $xy = 5$ | $y = \frac{5}{x}$ | inverse |
| **b.** $y = x - 4$ | $y = x - 4$ | neither |
| **c.** $\frac{y}{2} = x$ | $y = 2x$ | direct |

### Monitoring Progress

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**Tell whether $x$ and $y$ show *direct variation*, *inverse variation*, or *neither*.**

**1.** $6x = y$

**2.** $xy = -0.25$

**3.** $y + x = 10$

The general equation $y = ax$ for direct variation can be rewritten as $\frac{y}{x} = a$. So, a set of data pairs $(x, y)$ shows direct variation when the ratios $\frac{y}{x}$ are constant.

The general equation $y = \frac{a}{x}$ for inverse variation can be rewritten as $xy = a$. So, a set of data pairs $(x, y)$ shows inverse variation when the products $xy$ are constant.

## EXAMPLE 2 Classifying Data

Tell whether $x$ and $y$ show *direct variation*, *inverse variation*, or *neither*.

**a.**

| $x$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $y$ | $-12$ | $-6$ | $-4$ | $-3$ |

**b.**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 2 | 4 | 8 | 16 |

### SOLUTION

**a.** Find the products $xy$ and ratios $\frac{y}{x}$.

| $xy$ | $-24$ | $-24$ | $-24$ | $-24$ |
|---|---|---|---|---|
| $\frac{y}{x}$ | $\frac{-12}{2} = -6$ | $\frac{-6}{4} = -\frac{3}{2}$ | $\frac{-4}{6} = -\frac{2}{3}$ | $-\frac{3}{8}$ |

The products are constant.

The ratios are not constant.

▶ So, $x$ and $y$ show inverse variation.

**b.** Find the products $xy$ and ratios $\frac{y}{x}$.

| $xy$ | 2 | 8 | 24 | 64 |
|---|---|---|---|---|
| $\frac{y}{x}$ | $\frac{2}{1} = 2$ | $\frac{4}{2} = 2$ | $\frac{8}{3}$ | $\frac{16}{4} = 4$ |

The products are not constant.

The ratios are not constant.

▶ So, $x$ and $y$ show neither direct nor inverse variation.

**ANALYZING RELATIONSHIPS**

In Example 2(b), notice in the original table that as $x$ increases by 1, $y$ is multiplied by 2. So, the data in the table represent an exponential function.

## Monitoring Progress

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**Tell whether $x$ and $y$ show *direct variation*, *inverse variation*, or *neither*.**

**4.**

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ |
|---|---|---|---|---|
| $y$ | 20 | 15 | 10 | 5 |

**5.**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | 60 | 30 | 20 | 15 |

## Writing Inverse Variation Equations

## EXAMPLE 3 Writing an Inverse Variation Equation

The variables $x$ and $y$ vary inversely, and $y = 4$ when $x = 3$. Write an equation that relates $x$ and $y$. Then find $y$ when $x = -2$.

### SOLUTION

$y = \frac{a}{x}$ Write general equation for inverse variation.

$4 = \frac{a}{3}$ Substitute 4 for $y$ and 3 for $x$.

$12 = a$ Multiply each side by 3.

▶ The inverse variation equation is $y = \frac{12}{x}$. When $x = -2$, $y = \frac{12}{-2} = -6$.

**ANOTHER WAY**

Because $x$ and $y$ vary inversely, you also know that the products $xy$ are constant. This product equals the constant of variation $a$. So, you can quickly determine that $a = xy = 3(4) = 12$.

## EXAMPLE 4 Modeling with Mathematics

The time $t$ (in hours) that it takes a group of volunteers to build a playground varies inversely with the number $n$ of volunteers. It takes a group of 10 volunteers 8 hours to build the playground.

- Make a table showing the time that it would take to build the playground when the number of volunteers is 15, 20, 25, and 30.
- What happens to the time it takes to build the playground as the number of volunteers increases?

### SOLUTION

1. **Understand the Problem** You are given a description of two quantities that vary inversely and one pair of data values. You are asked to create a table that gives additional data pairs.

2. **Make a Plan** Use the time that it takes 10 volunteers to build the playground to find the constant of variation. Then write an inverse variation equation and substitute for the different numbers of volunteers to find the corresponding times.

3. **Solve the Problem**

$t = \frac{a}{n}$ Write general equation for inverse variation.

$8 = \frac{a}{10}$ Substitute 8 for $t$ and 10 for $n$.

$80 = a$ Multiply each side by 10.

The inverse variation equation is $t = \frac{80}{n}$. Make a table of values.

| $n$ | 15 | 20 | 25 | 30 |
|---|---|---|---|---|
| $t$ | $\frac{80}{15}$ = 5 h 20 min | $\frac{80}{20}$ = 4 h | $\frac{80}{25}$ = 3 h 12 min | $\frac{80}{30}$ = 2 h 40 min |

▶ As the number of volunteers increases, the time it takes to build the playground decreases.

4. **Look Back** Because the time decreases as the number of volunteers increases, the time for 5 volunteers to build the playground should be greater than 8 hours.

$$t = \frac{80}{5} = 16 \text{ hours } \checkmark$$

### LOOKING FOR A PATTERN

Notice that as the number of volunteers increases by 5, the time decreases by a lesser and lesser amount.

From $n = 15$ to $n = 20$, $t$ decreases by 1 hour 20 minutes.

From $n = 20$ to $n = 25$, $t$ decreases by 48 minutes.

From $n = 25$ to $n = 30$, $t$ decreases by 32 minutes.

## Monitoring Progress

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**The variables $x$ and $y$ vary inversely. Use the given values to write an equation relating $x$ and $y$. Then find $y$ when $x = 2$.**

**6.** $x = 4, y = 5$ **7.** $x = 6, y = -1$ **8.** $x = \frac{1}{2}, y = 16$

**9.** **WHAT IF?** In Example 4, it takes a group of 10 volunteers 12 hours to build the playground. How long would it take a group of 15 volunteers?

# 7.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **VOCABULARY** Explain how direct variation equations and inverse variation equations are different.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

| | |
|---|---|
| What is an inverse variation equation relating $x$ and $y$ with $a = 4$? | What is an equation for which the ratios $\frac{y}{x}$ are constant and $a = 4$? |
| What is an equation for which $y$ varies inversely with $x$ and $a = 4$? | What is an equation for which the products $xy$ are constant and $a = 4$? |

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–10, tell whether $x$ and $y$ show *direct variation*, *inverse variation*, or *neither*.** *(See Example 1.)*

3. $y = \frac{2}{x}$

4. $xy = 12$

5. $\frac{y}{x} = 8$

6. $4x = y$

7. $y = x + 4$

8. $x + y = 6$

9. $8y = x$

10. $xy = \frac{1}{5}$

**In Exercises 11–14, tell whether $x$ and $y$ show *direct variation*, *inverse variation*, or *neither*.** *(See Example 2.)*

11.

| $x$ | 12 | 18 | 23 | 29 | 34 |
|---|---|---|---|---|---|
| $y$ | 132 | 198 | 253 | 319 | 374 |

12.

| $x$ | 1.5 | 2.5 | 4 | 7.5 | 10 |
|---|---|---|---|---|---|
| $y$ | 13.5 | 22.5 | 36 | 67.5 | 90 |

13.

| $x$ | 4 | 6 | 8 | 8.4 | 12 |
|---|---|---|---|---|---|
| $y$ | 21 | 14 | 10.5 | 10 | 7 |

14.

| $x$ | 4 | 5 | 6.2 | 7 | 11 |
|---|---|---|---|---|---|
| $y$ | 16 | 11 | 10 | 9 | 6 |

**In Exercises 15–22, the variables $x$ and $y$ vary inversely. Use the given values to write an equation relating $x$ and $y$. Then find $y$ when $x = 3$.** *(See Example 3.)*

15. $x = 5, y = -4$

16. $x = 1, y = 9$

17. $x = -3, y = 8$

18. $x = 7, y = 2$

19. $x = \frac{3}{4}, y = 28$

20. $x = -4, y = -\frac{5}{4}$

21. $x = -12, y = -\frac{1}{6}$

22. $x = \frac{5}{3}, y = -7$

**ERROR ANALYSIS In Exercises 23 and 24, the variables $x$ and $y$ vary inversely. Describe and correct the error in writing an equation relating $x$ and $y$.**

23. $x = 8, y = 5$

$y = ax$

$5 = a(8)$

$\frac{5}{8} = a$

So, $y = \frac{5}{8}x$.

24. $x = 5, y = 2$

$xy = a$

$5 \cdot 2 = a$

$10 = a$

So, $y = 10x$.

**25.** **MODELING WITH MATHEMATICS** The number $y$ of songs that can be stored on an MP3 player varies inversely with the average size $x$ of a song. A certain MP3 player can store 2500 songs when the average size of a song is 4 megabytes (MB). *(See Example 4.)*

**a.** Make a table showing the numbers of songs that will fit on the MP3 player when the average size of a song is 2 MB, 2.5 MB, 3 MB, and 5 MB.

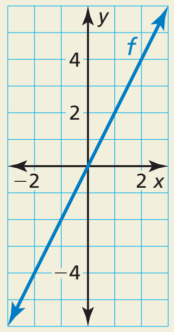
**b.** What happens to the number of songs as the average song size increases?

**26.** **MODELING WITH MATHEMATICS** When you stand on snow, the average pressure $P$ (in pounds per square inch) that you exert on the snow varies inversely with the total area $A$ (in square inches) of the soles of your footwear. Suppose the pressure is 0.43 pound per square inch when you wear the snowshoes shown. Write an equation that gives $P$ as a function of $A$. Then find the pressure when you wear the boots shown.


**27.** **PROBLEM SOLVING** Computer chips are etched onto silicon wafers. The table compares the area $A$ (in square millimeters) of a computer chip with the number $c$ of chips that can be obtained from a silicon wafer. Write a model that gives $c$ as a function of $A$. Then predict the number of chips per wafer when the area of a chip is 81 square millimeters.

| Area (mm$^2$), $A$ | 58 | 62 | 66 | 70 |
|---|---|---|---|---|
| Number of chips, $c$ | 448 | 424 | 392 | 376 |

**28.** **HOW DO YOU SEE IT?** Does the graph of $f$ represent inverse variation or direct variation? Explain your reasoning.



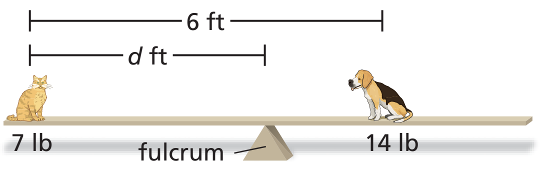

**29.** **MAKING AN ARGUMENT** You have enough money to buy 5 hats for \$10 each or 10 hats for \$5 each. Your friend says this situation represents inverse variation. Is your friend correct? Explain your reasoning.

**30.** **THOUGHT PROVOKING** The weight $w$ (in pounds) of an object varies inversely with the square of the distance $d$ (in miles) of the object from the center of Earth. At sea level (3978 miles from the center of the Earth), an astronaut weighs 210 pounds. How much does the astronaut weigh 200 miles above sea level?

**31.** **OPEN-ENDED** Describe a real-life situation that can be modeled by an inverse variation equation.

**32.** **CRITICAL THINKING** Suppose $x$ varies inversely with $y$ and $y$ varies inversely with $z$. How does $x$ vary with $z$? Justify your answer.

**33.** **USING STRUCTURE** To balance the board in the diagram, the distance (in feet) of each animal from the center of the board must vary inversely with its weight (in pounds). What is the distance of each animal from the fulcrum? Justify your answer.




## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Divide.** *(Section 4.3)*

**34.** $(x^2 + 2x - 99) \div (x + 11)$

**35.** $(3x^4 - 13x^2 - x^3 + 6x - 30) \div (3x^2 - x + 5)$

**Graph the function. Then state the domain and range.** *(Section 6.4)*

**36.** $f(x) = 5^x + 4$

**37.** $g(x) = e^{x-1}$

**38.** $y = \ln 3x - 6$

**39.** $h(x) = 2 \ln (x + 9)$

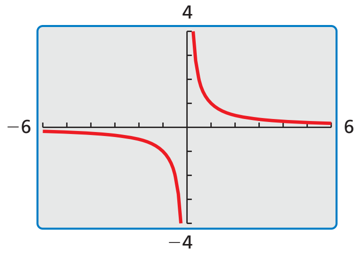
# 7.2 Graphing Rational Functions

## Essential Question

What are some of the characteristics of the graph of a rational function?

The parent function for rational functions with a linear numerator and a linear denominator is

$$f(x) = \frac{1}{x}. \qquad \text{Parent function}$$




The graph of this function, shown at the right, is a *hyperbola*.

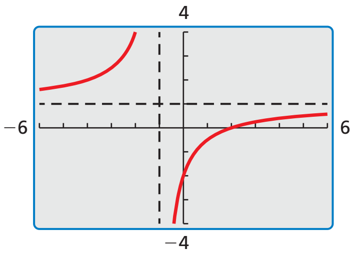
### EXPLORATION 1 Identifying Graphs of Rational Functions

**Work with a partner.** Each function is a transformation of the graph of the parent function $f(x) = \frac{1}{x}$. Match the function with its graph. Explain your reasoning. Then describe the transformation.

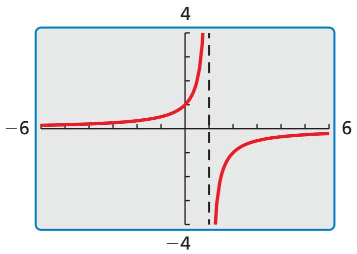
**a.** $g(x) = \dfrac{1}{x - 1}$ **b.** $g(x) = \dfrac{-1}{x - 1}$ **c.** $g(x) = \dfrac{x + 1}{x - 1}$

**d.** $g(x) = \dfrac{x - 2}{x + 1}$ **e.** $g(x) = \dfrac{x}{x + 2}$ **f.** $g(x) = \dfrac{-x}{x + 2}$

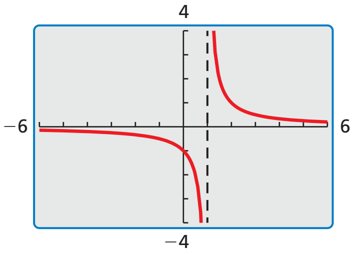
**A.**



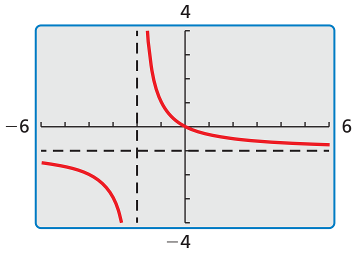

**B.**



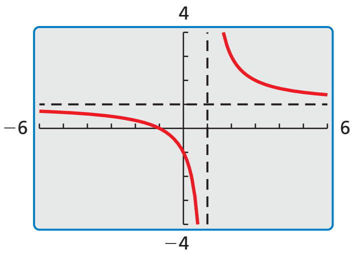

**C.**



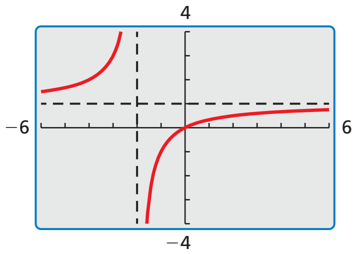

**D.**




**E.**




**F.**




**LOOKING FOR STRUCTURE**

To be proficient in math, you need to look closely to discern a pattern or structure.

## Communicate Your Answer

**2.** What are some of the characteristics of the graph of a rational function?

**3.** Determine the intercepts, asymptotes, domain, and range of the rational function $g(x) = \dfrac{x - a}{x - b}$.

# 7.2 Lesson

## Core Vocabulary

rational function, *p. 366*

***Previous***
domain
range
asymptote
long division

## What You Will Learn

- Graph simple rational functions.
- Translate simple rational functions.
- Graph other rational functions.

## Graphing Simple Rational Functions

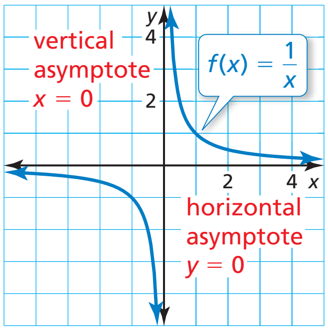
A **rational function** has the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$. The inverse variation function $f(x) = \frac{a}{x}$ is a rational function. The graph of this function when $a = 1$ is shown below.

## Core Concept

### Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \frac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called branches. The domain and range are all nonzero real numbers.

Any function of the form $g(x) = \frac{a}{x}$ $(a \neq 0)$ has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.




### STUDY TIP

Notice that $\frac{1}{x} \to 0$ as $x \to \infty$ and as $x \to -\infty$. This explains why $y = 0$ is a horizontal asymptote of the graph of $f(x) = \frac{1}{x}$. You can also analyze $y$-values as $x$ approaches 0 to see why $x = 0$ is a vertical asymptote.

### EXAMPLE 1 Graphing a Rational Function of the Form $y = \frac{a}{x}$

Graph $g(x) = \frac{4}{x}$. Compare the graph with the graph of $f(x) = \frac{1}{x}$.

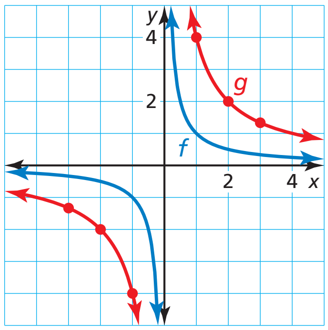
**SOLUTION**

**Step 1** The function is of the form $g(x) = \frac{a}{x}$, so the asymptotes are $x = 0$ and $y = 0$. Draw the asymptotes.

**Step 2** Make a table of values and plot the points. Include both positive and negative values of $x$.

| $x$ | $-3$ | $-2$ | $-1$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $y$ | $-\frac{4}{3}$ | $-2$ | $-4$ | $4$ | $2$ | $\frac{4}{3}$ |

**Step 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.




The graph of $g$ lies farther from the axes than the graph of $f$. Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.

### LOOKING FOR STRUCTURE

Because the function is of the form $g(x) = a \cdot f(x)$, where $a = 4$, the graph of $g$ is a vertical stretch by a factor of 4 of the graph of $f$.

## Monitoring Progress

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1. Graph $g(x) = \frac{-6}{x}$. Compare the graph with the graph of $f(x) = \frac{1}{x}$.

## Translating Simple Rational Functions

### Core Concept

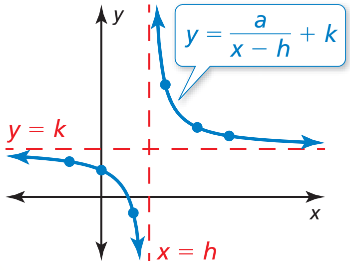
**Graphing Translations of Simple Rational Functions**

To graph a rational function of the form $y = \dfrac{a}{x-h} + k$, follow these steps:

**Step 1** Draw the asymptotes $x = h$ and $y = k$.

**Step 2** Plot points to the left and to the right of the vertical asymptote.

**Step 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.




**EXAMPLE 2** **Graphing a Translation of a Rational Function**

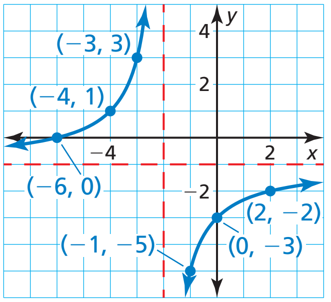
Graph $g(x) = \dfrac{-4}{x+2} - 1$. State the domain and range.

**SOLUTION**

**Step 1** Draw the asymptotes $x = -2$ and $y = -1$.

**Step 2** Plot points to the left of the vertical asymptote, such as $(-3, 3)$, $(-4, 1)$, and $(-6, 0)$. Plot points to the right of the vertical asymptote, such as $(-1, -5)$, $(0, -3)$, and $(2, -2)$.

**Step 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.




The domain is all real numbers except $-2$ and the range is all real numbers except $-1$.

**LOOKING FOR STRUCTURE**

Let $f(x) = \dfrac{-4}{x}$. Notice that $g$ is of the form $g(x) = f(x-h) + k$, where $h = -2$ and $k = -1$. So, the graph of $g$ is a translation 2 units left and 1 unit down of the graph of $f$.

### Monitoring Progress

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**Graph the function. State the domain and range.**

**2.** $y = \dfrac{3}{x} - 2$

**3.** $y = \dfrac{-1}{x+4}$

**4.** $y = \dfrac{1}{x-1} + 5$

## Graphing Other Rational Functions

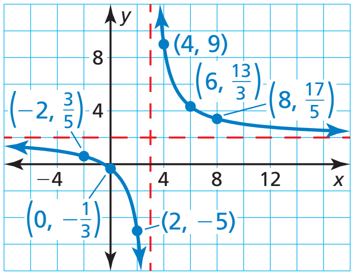
All rational functions of the form $y = \dfrac{ax+b}{cx+d}$ also have graphs that are hyperbolas.

- The vertical asymptote of the graph is the line $x = -\dfrac{d}{c}$ because the function is undefined when the denominator $cx + d$ is zero.
- The horizontal asymptote is the line $y = \dfrac{a}{c}$.

## EXAMPLE 3 Graphing a Rational Function of the Form $y = \dfrac{ax + b}{cx + d}$

Graph $f(x) = \dfrac{2x + 1}{x - 3}$. State the domain and range.

### SOLUTION




**Step 1** Draw the asymptotes. Solve $x - 3 = 0$ for $x$ to find the vertical asymptote $x = 3$. The horizontal asymptote is the line $y = \frac{a}{c} = \frac{2}{1} = 2$.

**Step 2** Plot points to the left of the vertical asymptote, such as $(2, -5)$, $\left(0, -\frac{1}{3}\right)$, and $\left(-2, \frac{3}{5}\right)$. Plot points to the right of the vertical asymptote, such as $(4, 9)$, $\left(6, \frac{13}{3}\right)$, and $\left(8, \frac{17}{5}\right)$.

**Step 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

▶ The domain is all real numbers except 3 and the range is all real numbers except 2.

Rewriting a rational function may reveal properties of the function and its graph. For example, rewriting a rational function in the form $y = \frac{a}{x - h} + k$ reveals that it is a translation of $y = \frac{a}{x}$ with vertical asymptote $x = h$ and horizontal asymptote $y = k$.

## EXAMPLE 4 Rewriting and Graphing a Rational Function

Rewrite $g(x) = \dfrac{3x + 5}{x + 1}$ in the form $g(x) = \dfrac{a}{x - h} + k$. Graph the function. Describe the graph of $g$ as a transformation of the graph of $f(x) = \dfrac{a}{x}$.

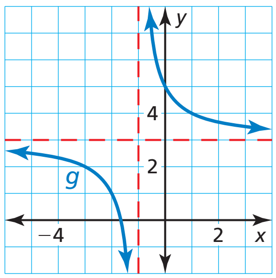
**ANOTHER WAY**

You will use a different method to rewrite $g$ in Example 5 of Lesson 7.4.

### SOLUTION

Rewrite the function by using long division:

$$\begin{array}{r} 3 \\ x + 1 \enclose{longdiv}{3x + 5} \\ \underline{3x + 3} \\ 2 \end{array}$$




▶ The rewritten function is $g(x) = \dfrac{2}{x + 1} + 3$. The graph of $g$ is a translation 1 unit left and 3 units up of the graph of $f(x) = \dfrac{2}{x}$.

## Monitoring Progress

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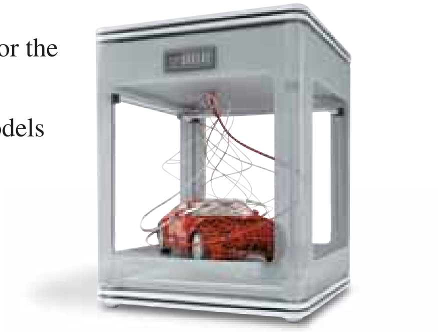
**Graph the function. State the domain and range.**

**5.** $f(x) = \dfrac{x - 1}{x + 3}$ **6.** $f(x) = \dfrac{2x + 1}{4x - 2}$ **7.** $f(x) = \dfrac{-3x + 2}{-x - 1}$

**8.** Rewrite $g(x) = \dfrac{2x + 3}{x + 1}$ in the form $g(x) = \dfrac{a}{x - h} + k$. Graph the function. Describe the graph of $g$ as a transformation of the graph of $f(x) = \dfrac{a}{x}$.

## EXAMPLE 5 Modeling with Mathematics

A 3-D printer builds up layers of materials to make three-dimensional models. Each deposited layer bonds to the layer below it. A company decides to make small display models of engine components using a 3-D printer. The printer costs \$1000. The material for each model costs \$50.



- Estimate how many models must be printed for the average cost per model to fall to \$90.
- What happens to the average cost as more models are printed?

### SOLUTION

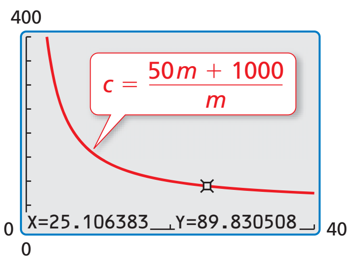
1. **Understand the Problem** You are given the cost of a printer and the cost to create a model using the printer. You are asked to find the number of models for which the average cost falls to \$90.

2. **Make a Plan** Write an equation that represents the average cost. Use a graphing calculator to estimate the number of models for which the average cost is about \$90. Then analyze the horizontal asymptote of the graph to determine what happens to the average cost as more models are printed.

3. **Solve the Problem** Let $c$ be the average cost (in dollars) and $m$ be the number of models printed.

$$c = \frac{(\text{Unit cost})(\text{Number printed}) + (\text{Cost of printer})}{\text{Number printed}} = \frac{50m + 1000}{m}$$

Use a graphing calculator to graph the function.




▶ Using the *trace* feature, the average cost falls to \$90 per model after about 25 models are printed. Because the horizontal asymptote is $c = 50$, the average cost approaches \$50 as more models are printed.

4. **Look Back** Use a graphing calculator to create tables of values for large values of $m$. The tables show that the average cost approaches \$50 as more models are printed.

| X | Y1 |
|---|---|
| 0 | ERROR |
| 50 | 70 |
| 100 | 60 |
| 150 | 56.667 |
| 200 | 55 |
| 250 | 54 |
| 300 | 53.333 |

X=0

| X | Y1 |
|---|---|
| 0 | ERROR |
| 10000 | 50.1 |
| 20000 | 50.05 |
| 30000 | 50.033 |
| 40000 | 50.025 |
| 50000 | 50.02 |
| 60000 | 50.017 |

X=0

### USING A GRAPHING CALCULATOR

Because the number of models and average cost cannot be negative, choose a viewing window in the first quadrant.

## Monitoring Progress

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9. **WHAT IF?** How do the answers in Example 5 change when the cost of the 3-D printer is \$800?

## Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The function $y = \frac{7}{x+4} + 3$ has a(n) __________ of all real numbers except 3 and a(n) __________ of all real numbers except $-4$.

2. **WRITING** Is $f(x) = \frac{-3x+5}{2^x+1}$ a rational function? Explain your reasoning.

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–10, graph the function. Compare the graph with the graph of $f(x) = \frac{1}{x}$.** *(See Example 1.)*

3. $g(x) = \frac{3}{x}$

4. $g(x) = \frac{10}{x}$

5. $g(x) = \frac{-5}{x}$

6. $g(x) = \frac{-9}{x}$

7. $g(x) = \frac{15}{x}$

8. $g(x) = \frac{-12}{x}$

9. $g(x) = \frac{-0.5}{x}$

10. $g(x) = \frac{0.1}{x}$

**In Exercises 11–18, graph the function. State the domain and range.** *(See Example 2.)*

11. $g(x) = \frac{4}{x} + 3$

12. $y = \frac{2}{x} - 3$

13. $h(x) = \frac{6}{x-1}$

14. $y = \frac{1}{x+2}$

15. $h(x) = \frac{-3}{x+2}$

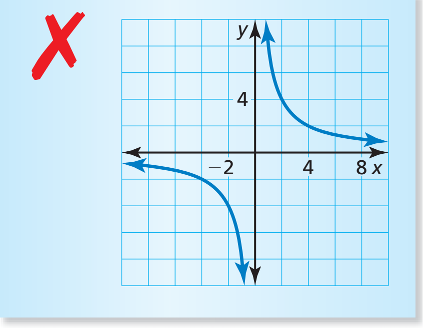
16. $f(x) = \frac{-2}{x-7}$

17. $g(x) = \frac{-3}{x-4} - 1$

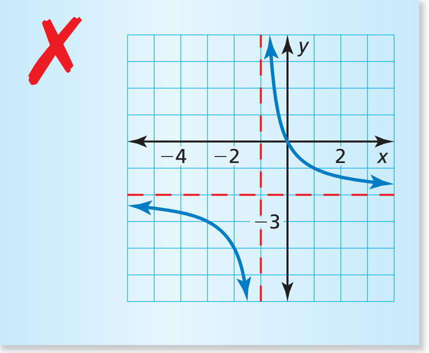
18. $y = \frac{10}{x+7} - 5$

**ERROR ANALYSIS** **In Exercises 19 and 20, describe and correct the error in graphing the rational function.**

19. $y = \frac{-8}{x}$




20. $y = \frac{2}{x-1} - 2$



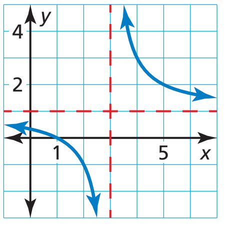

**ANALYZING RELATIONSHIPS** **In Exercises 21–24, match the function with its graph. Explain your reasoning.**

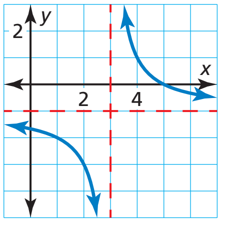
21. $g(x) = \frac{2}{x-3} + 1$

22. $h(x) = \frac{2}{x+3} + 1$

23. $f(x) = \frac{2}{x-3} - 1$

24. $y = \frac{2}{x+3} - 1$

A.



B.
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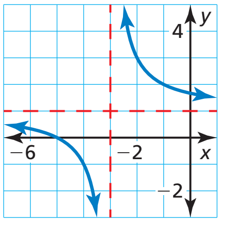
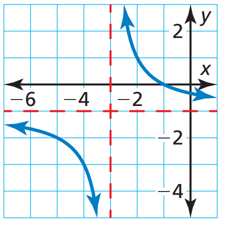



D.


**In Exercises 25–32, graph the function. State the domain and range.** *(See Example 3.)*

**25.** $f(x) = \dfrac{x+4}{x-3}$

**26.** $y = \dfrac{x-1}{x+5}$

**27.** $y = \dfrac{x+6}{4x-8}$

**28.** $h(x) = \dfrac{8x+3}{2x-6}$

**29.** $f(x) = \dfrac{-5x+2}{4x+5}$

**30.** $g(x) = \dfrac{6x-1}{3x-1}$

**31.** $h(x) = \dfrac{-5x}{-2x-3}$

**32.** $y = \dfrac{-2x+3}{-x+10}$

**In Exercises 33–40, rewrite the function in the form $g(x) = \dfrac{a}{x-h} + k$. Graph the function. Describe the graph of $g$ as a transformation of the graph of $f(x) = \dfrac{a}{x}$.** *(See Example 4.)*

**33.** $g(x) = \dfrac{5x+6}{x+1}$

**34.** $g(x) = \dfrac{7x+4}{x-3}$

**35.** $g(x) = \dfrac{2x-4}{x-5}$

**36.** $g(x) = \dfrac{4x-11}{x-2}$

**37.** $g(x) = \dfrac{x+18}{x-6}$

**38.** $g(x) = \dfrac{x+2}{x-8}$

**39.** $g(x) = \dfrac{7x-20}{x+13}$

**40.** $g(x) = \dfrac{9x-3}{x+7}$

**41. PROBLEM SOLVING** Your school purchases a math software program. The program has an initial cost of \$500 plus \$20 for each student that uses the program. *(See Example 5.)*

**a.** Estimate how many students must use the program for the average cost per student to fall to \$30.

**b.** What happens to the average cost as more students use the program?

**42. PROBLEM SOLVING** To join a rock climbing gym, you must pay an initial fee of \$100 and a monthly fee of \$59.

**a.** Estimate how many months you must purchase a membership for the average cost per month to fall to \$69.

**b.** What happens to the average cost as the number of months that you are a member increases?

**43. USING STRUCTURE** What is the vertical asymptote of the graph of the function $y = \dfrac{2}{x+4} + 7$?

Ⓐ $x = -7$ Ⓑ $x = -4$

Ⓒ $x = 4$ Ⓓ $x = 7$

**44. REASONING** What are the $x$-intercept(s) of the graph of the function $y = \dfrac{x-5}{x^2-1}$?

Ⓐ $1, -1$ Ⓑ $5$

Ⓒ $1$ Ⓓ $-5$

**45. USING TOOLS** The time $t$ (in seconds) it takes for sound to travel 1 kilometer can be modeled by

$$t = \frac{1000}{0.6T + 331}$$

where $T$ is the air temperature (in degrees Celsius).

**a.** You are 1 kilometer from a lightning strike. You hear the thunder 2.9 seconds later. Use a graph to find the approximate air temperature.

**b.** Find the average rate of change in the time it takes sound to travel 1 kilometer as the air temperature increases from 0°C to 10°C.

**46. MODELING WITH MATHEMATICS** A business is studying the cost to remove a pollutant from the ground at its site. The function $y = \dfrac{15x}{1.1 - x}$ models the estimated cost $y$ (in thousands of dollars) to remove $x$ percent (expressed as a decimal) of the pollutant.

**a.** Graph the function. Describe a reasonable domain and range.

**b.** How much does it cost to remove 20% of the pollutant? 40% of the pollutant? 80% of the pollutant? Does doubling the percentage of the pollutant removed double the cost? Explain.

**USING TOOLS In Exercises 47–50, use a graphing calculator to graph the function. Then determine whether the function is *even*, *odd*, or *neither*.**

**47.** $h(x) = \dfrac{6}{x^2+1}$

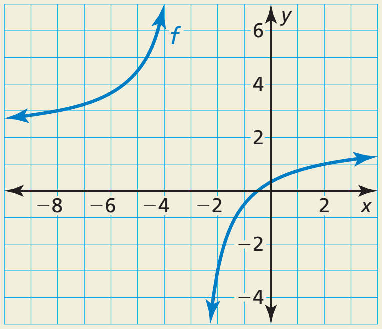
**48.** $f(x) = \dfrac{2x^2}{x^2-9}$

**49.** $y = \dfrac{x^3}{3x^2 + x^4}$

**50.** $f(x) = \dfrac{4x^2}{2x^3 - x}$

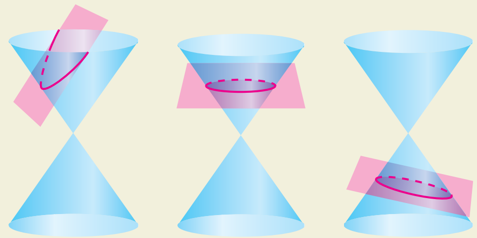
**51. MAKING AN ARGUMENT** Your friend claims it is possible for a rational function to have two vertical asymptotes. Is your friend correct? Justify your answer.

**52. HOW DO YOU SEE IT?** Use the graph of $f$ to determine the equations of the asymptotes. Explain.




**53. DRAWING CONCLUSIONS** In what line(s) is the graph of $y = \frac{1}{x}$ symmetric? What does this symmetry tell you about the inverse of the function $f(x) = \frac{1}{x}$?

**54. THOUGHT PROVOKING** There are four basic types of conic sections: parabola, circle, ellipse, and hyperbola. Each of these can be represented by the intersection of a double-napped cone and a plane. The intersections for a parabola, circle, and ellipse are shown below. Sketch the intersection for a hyperbola.



Parabola Circle Ellipse

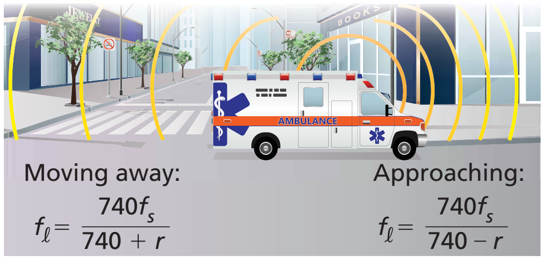
**55. REASONING** The graph of the rational function $f$ is a hyperbola. The asymptotes of the graph of $f$ intersect at (3, 2). The point (2, 1) is on the graph. Find another point on the graph. Explain your reasoning.

**56. ABSTRACT REASONING** Describe the intervals where the graph of $y = \frac{a}{x}$ is increasing or decreasing when (a) $a > 0$ and (b) $a < 0$. Explain your reasoning.

**57. PROBLEM SOLVING** An Internet service provider charges a \$50 installation fee and a monthly fee of \$43. The table shows the average monthly costs $y$ of a competing provider for $x$ months of service. Under what conditions would a person choose one provider over the other? Explain your reasoning.

| Months, $x$ | Average monthly cost (dollars), $y$ |
|---|---|
| 6 | \$49.83 |
| 12 | \$46.92 |
| 18 | \$45.94 |
| 24 | \$45.45 |

**58. MODELING WITH MATHEMATICS** The Doppler effect occurs when the source of a sound is moving relative to a listener, so that the frequency $f_\ell$ (in hertz) heard by the listener is different from the frequency $f_s$ (in hertz) at the source. In both equations below, $r$ is the speed (in miles per hour) of the sound source.




**a.** An ambulance siren has a frequency of 2000 hertz. Write two equations modeling the frequencies heard when the ambulance is approaching and when the ambulance is moving away.

**b.** Graph the equations in part (a) using the domain $0 \le r \le 60$.

**c.** For any speed $r$, how does the frequency heard for an approaching sound source compare with the frequency heard when the source moves away?

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Factor the polynomial.** *(Skills Review Handbook)*

**59.** $4x^2 - 4x - 80$ **60.** $3x^2 - 3x - 6$ **61.** $2x^2 - 2x - 12$ **62.** $10x^2 + 31x - 14$

**Simplify the expression.** *(Section 5.2)*

**63.** $3^2 \cdot 3^4$ **64.** $2^{1/2} \cdot 2^{3/5}$ **65.** $\frac{6^{5/6}}{6^{1/6}}$ **66.** $\frac{6^8}{6^{10}}$

# 7.1–7.2 What Did You Learn?

## Core Vocabulary

inverse variation, *p. 360*
constant of variation, *p. 360*
rational function, *p. 366*

## Core Concepts

### Section 7.1

Inverse Variation, *p. 360*
Writing Inverse Variation Equations, *p. 361*

### Section 7.2

Parent Function for Simple Rational Functions, *p. 366*
Graphing Translations of Simple Rational Functions, *p. 367*

## Mathematical Practices

1. Explain the meaning of the given information in Exercise 25 on page 364.
2. How are you able to recognize whether the logic used in Exercise 29 on page 364 is correct or flawed?
3. How can you evaluate the reasonableness of your answer in part (b) of Exercise 41 on page 371?
4. How did the context allow you to determine a reasonable domain and range for the function in Exercise 46 on page 371?

Study Skills

## Analyzing Your Errors

### Study Errors

**What Happens:** You do not study the right material or you do not learn it well enough to remember it on a test without resources such as notes.

**How to Avoid This Error:** Take a practice test. Work with a study group. Discuss the topics on the test with your teacher. Do not try to learn a whole chapter's worth of material in one night.

# 7.1–7.2 Quiz

**Tell whether $x$ and $y$ show *direct variation*, *inverse variation*, or *neither*. Explain your reasoning.** *(Section 7.1)*

**1.** $x + y = 7$

**2.** $\frac{2}{5}x = y$

**3.** $xy = 0.45$

**4.**

| $x$ | 3 | 6 | 9 | 12 |
|---|---|---|---|---|
| $y$ | 9 | 18 | 27 | 36 |

**5.**

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | $-24$ | $-12$ | $-8$ | $-6$ |

**6.**

| $x$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $y$ | 72 | 36 | 18 | 9 |

**7.** The variables $x$ and $y$ vary inversely, and $y = 10$ when $x = 5$. Write an equation that relates $x$ and $y$. Then find $y$ when $x = -2$. *(Section 7.1)*

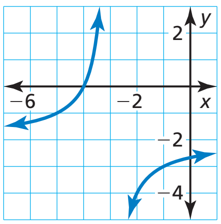
**Match the equation with the correct graph. Explain your reasoning.** *(Section 7.2)*

**8.** $f(x) = \dfrac{3}{x} + 2$

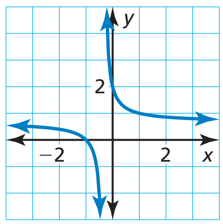
**9.** $y = \dfrac{-2}{x + 3} - 2$

**10.** $h(x) = \dfrac{2x + 2}{3x + 1}$

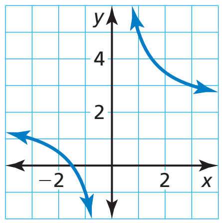
**A.**




**B.**




**C.**



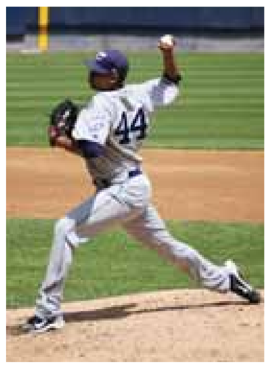

**11.** Rewrite $g(x) = \dfrac{2x + 9}{x + 8}$ in the form $g(x) = \dfrac{a}{x - h} + k$. Graph the function. Describe the graph of $g$ as a transformation of the graph of $f(x) = \dfrac{a}{x}$. *(Section 7.2)*

**12.** The time $t$ (in minutes) required to empty a tank varies inversely with the pumping rate $r$ (in gallons per minute). The rate of a certain pump is 70 gallons per minute. It takes the pump 20 minutes to empty the tank. Complete the table for the times it takes the pump to empty a tank for the given pumping rates. *(Section 7.1)*

| Pumping rate (gal/min) | Time (min) |
|---|---|
| 50 | |
| 60 | |
| 65 | |
| 70 | |

**13.** A pitcher throws 16 strikes in the first 38 pitches. The table shows how a pitcher's strike percentage changes when the pitcher throws $x$ consecutive strikes after the first 38 pitches. Write a rational function for the strike percentage in terms of $x$. Graph the function. How many consecutive strikes must the pitcher throw to reach a strike percentage of 0.60? *(Section 7.2)*

| $x$ | Total strikes | Total pitches | Strike percentage |
|---|---|---|---|
| 0 | 16 | 38 | 0.42 |
| 5 | 21 | 43 | 0.49 |
| 10 | 26 | 48 | 0.54 |
| $x$ | $x + 16$ | $x + 38$ | |



# 7.3 Multiplying and Dividing Rational Expressions

**Essential Question** How can you determine the excluded values in a product or quotient of two rational expressions?

You can multiply and divide rational expressions in much the same way that you multiply and divide fractions. Values that make the denominator of an expression zero are *excluded values*.

$$\frac{1}{\cancel{x}} \cdot \frac{\cancel{x}}{x+1} = \frac{1}{x+1},\ x \neq 0$$ Product of rational expressions

$$\frac{1}{x} \div \frac{x}{x+1} = \frac{1}{x} \cdot \frac{x+1}{x} = \frac{x+1}{x^2},\ x \neq -1$$ Quotient of rational expressions

## EXPLORATION 1 Multiplying and Dividing Rational Expressions

**Work with a partner.** Find the product or quotient of the two rational expressions. Then match the product or quotient with its excluded values. Explain your reasoning.

| Product or Quotient | Excluded Values |
|---|---|
| **a.** $\frac{1}{x-1} \cdot \frac{x-2}{x+1} =$ | **A.** $-1$, 0, and 2 |
| **b.** $\frac{1}{x-1} \cdot \frac{-1}{x-1} =$ | **B.** $-2$ and 1 |
| **c.** $\frac{1}{x-2} \cdot \frac{x-2}{x+1} =$ | **C.** $-2$, 0, and 1 |
| **d.** $\frac{x+2}{x-1} \cdot \frac{-x}{x+2} =$ | **D.** $-1$ and 2 |
| **e.** $\frac{x}{x+2} \div \frac{x+1}{x+2} =$ | **E.** $-1$, 0, and 1 |
| **f.** $\frac{x}{x-2} \div \frac{x+1}{x} =$ | **F.** $-1$ and 1 |
| **g.** $\frac{x}{x+2} \div \frac{x}{x-1} =$ | **G.** $-2$ and $-1$ |
| **h.** $\frac{x+2}{x} \div \frac{x+1}{x-1} =$ | **H.** 1 |

**REASONING ABSTRACTLY**

To be proficient in math, you need to know and flexibly use different properties of operations and objects.

## EXPLORATION 2 Writing a Product or Quotient

**Work with a partner.** Write a product or quotient of rational expressions that has the given excluded values. Justify your answer.

**a.** $-1$ **b.** $-1$ and 3 **c.** $-1$, 0, and 3

## Communicate Your Answer

**3.** How can you determine the excluded values in a product or quotient of two rational expressions?

**4.** Is it possible for the product or quotient of two rational expressions to have *no* excluded values? Explain your reasoning. If it is possible, give an example.

# 7.3 Lesson

## Core Vocabulary

rational expression, *p. 376*
simplified form of a rational expression, *p. 376*

***Previous***
fractions
polynomials
domain
equivalent expressions
reciprocal

## What You Will Learn

- Simplify rational expressions.
- Multiply rational expressions.
- Divide rational expressions.

## Simplifying Rational Expressions

A **rational expression** is a fraction whose numerator and denominator are nonzero polynomials. The *domain* of a rational expression excludes values that make the denominator zero. A rational expression is in **simplified form** when its numerator and denominator have no common factors (other than $\pm 1$).

## Core Concept

### Simplifying Rational Expressions

Let $a$, $b$, and $c$ be expressions with $b \neq 0$ and $c \neq 0$.

**Property** $\dfrac{a\cancel{c}}{b\cancel{c}} = \dfrac{a}{b}$ Divide out common factor $c$.

**Examples** $\dfrac{15}{65} = \dfrac{3 \cdot \cancel{5}}{13 \cdot \cancel{5}} = \dfrac{3}{13}$ Divide out common factor 5.

$\dfrac{4\cancel{(x+3)}}{(x+3)\cancel{(x+3)}} = \dfrac{4}{x+3}$ Divide out common factor $x + 3$.

### STUDY TIP

Notice that you can divide out common factors in the second expression at the right. You cannot, however, divide out like terms in the third expression.

Simplifying a rational expression usually requires two steps. First, factor the numerator and denominator. Then, divide out any factors that are common to both the numerator and denominator. Here is an example:

$$\frac{x^2 + 7x}{x^2} = \frac{x(x+7)}{x \cdot x} = \frac{x+7}{x}$$

### EXAMPLE 1 Simplifying a Rational Expression

Simplify $\dfrac{x^2 - 4x - 12}{x^2 - 4}$.

**SOLUTION**

$\dfrac{x^2 - 4x - 12}{x^2 - 4} = \dfrac{(x+2)(x-6)}{(x+2)(x-2)}$ Factor numerator and denominator.

$= \dfrac{\cancel{(x+2)}(x-6)}{\cancel{(x+2)}(x-2)}$ Divide out common factor.

$= \dfrac{x-6}{x-2}, \quad x \neq -2$ Simplified form

The original expression is undefined when $x = -2$. To make the original and simplified expressions equivalent, restrict the domain of the simplified expression by excluding $x = -2$. Both expressions are undefined when $x = 2$, so it is not necessary to list it.

### COMMON ERROR

Do not divide out variable terms that are not factors.

$\dfrac{x - 6}{x - 2} \neq \dfrac{-6}{-2}$

## Monitoring Progress

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**Simplify the rational expression, if possible.**

**1.** $\dfrac{2(x+1)}{(x+1)(x+3)}$ **2.** $\dfrac{x+4}{x^2 - 16}$ **3.** $\dfrac{4}{x(x+2)}$ **4.** $\dfrac{x^2 - 2x - 3}{x^2 - x - 6}$

## Multiplying Rational Expressions

The rule for multiplying rational expressions is the same as the rule for multiplying numerical fractions: multiply numerators, multiply denominators, and write the new fraction in simplified form. Similar to rational numbers, rational expressions are closed under multiplication.

### Core Concept

**Multiplying Rational Expressions**

Let $a$, $b$, $c$, and $d$ be expressions with $b \neq 0$ and $d \neq 0$.

**Property** $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ Simplify $\frac{ac}{bd}$ if possible.

**Example** $\frac{5x^2}{2xy^2} \cdot \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{\cancel{10} \cdot 3 \cdot \cancel{x} \cdot x^2 \cdot \cancel{y^3}}{\cancel{10} \cdot 2 \cdot \cancel{x} \cdot \cancel{y^3}} = \frac{3x^2}{2}, \quad x \neq 0, y \neq 0$

**ANOTHER WAY**

In Example 2, you can first simplify each rational expression, then multiply, and finally simplify the result.

$$\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$$
$$= \frac{4x^2}{y} \cdot \frac{7x^4y^2}{4}$$
$$= \frac{\cancel{4} \cdot 7 \cdot x^6 \cdot \cancel{y} \cdot y}{\cancel{4} \cdot \cancel{y}}$$
$$= 7x^6y, \quad x \neq 0, y \neq 0$$

### EXAMPLE 2 Multiplying Rational Expressions

Find the product $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$.

**SOLUTION**

$\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y} = \frac{56x^7y^4}{8xy^3}$ Multiply numerators and denominators.

$= \frac{\cancel{8} \cdot 7 \cdot \cancel{x} \cdot x^6 \cdot \cancel{y^3} \cdot y}{\cancel{8} \cdot \cancel{x} \cdot \cancel{y^3}}$ Factor and divide out common factors.

$= 7x^6y, \quad x \neq 0, y \neq 0$ Simplified form

### EXAMPLE 3 Multiplying Rational Expressions

Find the product $\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x}$.

**SOLUTION**

$\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x} = \frac{3x(1 - x)}{(x - 1)(x + 5)} \cdot \frac{(x + 5)(x - 4)}{3x}$ Factor numerators and denominators.

$= \frac{3x(1 - x)(x + 5)(x - 4)}{(x - 1)(x + 5)(3x)}$ Multiply numerators and denominators.

$= \frac{3x(-1)(x - 1)(x + 5)(x - 4)}{(x - 1)(x + 5)(3x)}$ Rewrite $1 - x$ as $(-1)(x - 1)$.

$= \frac{\cancel{3x}(-1)\cancel{(x - 1)}\cancel{(x + 5)}(x - 4)}{\cancel{(x - 1)}\cancel{(x + 5)}\cancel{(3x)}}$ Divide out common factors.

$= -x + 4, \quad x \neq -5, x \neq 0, x \neq 1$ Simplified form

**Check**

| X | Y1 | Y2 |
|---|---|---|
| −5 | ERROR | 9 |
| −4 | 8 | 8 |
| −3 | 7 | 7 |
| −2 | 6 | 6 |
| −1 | 5 | 5 |
| 0 | ERROR | 4 |
| 1 | ERROR | 3 |

X=−4

Check the simplified expression. Enter the original expression as $y_1$ and the simplified expression as $y_2$ in a graphing calculator. Then use the *table* feature to compare the values of the two expressions. The values of $y_1$ and $y_2$ are the same, except when $x = -5$, $x = 0$, and $x = 1$. So, when these values are excluded from the domain of the simplified expression, it is equivalent to the original expression.

## EXAMPLE 4 Multiplying a Rational Expression by a Polynomial

**STUDY TIP**

Notice that $x^2 + 3x + 9$ does not equal zero for any real value of $x$. So, no values must be excluded from the domain to make the simplified form equivalent to the original.

Find the product $\dfrac{x+2}{x^3-27} \cdot (x^2 + 3x + 9)$.

**SOLUTION**

$$\frac{x+2}{x^3-27} \cdot (x^2+3x+9) = \frac{x+2}{x^3-27} \cdot \frac{x^2+3x+9}{1}$$ Write polynomial as a rational expression.

$$= \frac{(x+2)(x^2+3x+9)}{(x-3)(x^2+3x+9)}$$ Multiply. Factor denominator.

$$= \frac{(x+2)\cancel{(x^2+3x+9)}}{(x-3)\cancel{(x^2+3x+9)}}$$ Divide out common factors.

$$= \frac{x+2}{x-3}$$ Simplified form

### Monitoring Progress

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**Find the product.**

**5.** $\dfrac{3x^5y^2}{8xy} \cdot \dfrac{6xy^2}{9x^3y}$

**6.** $\dfrac{2x^2-10x}{x^2-25} \cdot \dfrac{x+3}{2x^2}$

**7.** $\dfrac{x+5}{x^3-1} \cdot (x^2+x+1)$

## Dividing Rational Expressions

To divide one rational expression by another, multiply the first rational expression by the reciprocal of the second rational expression. Rational expressions are closed under nonzero division.

### Core Concept

**Dividing Rational Expressions**

Let $a$, $b$, $c$, and $d$ be expressions with $b \neq 0$, $c \neq 0$, and $d \neq 0$.

**Property** $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}$ Simplify $\dfrac{ad}{bc}$ if possible.

**Example** $\dfrac{7}{x+1} \div \dfrac{x+2}{2x-3} = \dfrac{7}{x+1} \cdot \dfrac{2x-3}{x+2} = \dfrac{7(2x-3)}{(x+1)(x+2)},\ x \neq \dfrac{3}{2}$

## EXAMPLE 5 Dividing Rational Expressions

Find the quotient $\dfrac{7x}{2x-10} \div \dfrac{x^2-6x}{x^2-11x+30}$.

**SOLUTION**

$$\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30} = \frac{7x}{2x-10} \cdot \frac{x^2-11x+30}{x^2-6x}$$ Multiply by reciprocal.

$$= \frac{7x}{2(x-5)} \cdot \frac{(x-5)(x-6)}{x(x-6)}$$ Factor.

$$= \frac{7\cancel{x}\cancel{(x-5)}\cancel{(x-6)}}{2\cancel{(x-5)}\cancel{(x)}\cancel{(x-6)}}$$ Multiply. Divide out common factors.

$$= \frac{7}{2},\quad x \neq 0, x \neq 5, x \neq 6$$ Simplified form

## EXAMPLE 6 Dividing a Rational Expression by a Polynomial

Find the quotient $\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$.

### SOLUTION

$$\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x) = \frac{6x^2 + x - 15}{4x^2} \cdot \frac{1}{3x^2 + 5x} \qquad \text{Multiply by reciprocal.}$$

$$= \frac{(3x + 5)(2x - 3)}{4x^2} \cdot \frac{1}{x(3x + 5)} \qquad \text{Factor.}$$

$$= \frac{\cancel{(3x + 5)}(2x - 3)}{4x^2(x)\cancel{(3x + 5)}} \qquad \text{Divide out common factors.}$$

$$= \frac{2x - 3}{4x^3},\quad x \neq -\frac{5}{3} \qquad \text{Simplified form}$$

## EXAMPLE 7 Solving a Real-Life Problem

The total annual amount $I$ (in millions of dollars) of personal income earned in Alabama and its annual population $P$ (in millions) can be modeled by

$$I = \frac{6922t + 106{,}947}{0.0063t + 1}$$

and

$$P = 0.0343t + 4.432$$

where $t$ represents the year, with $t = 1$ corresponding to 2001. Find a model $M$ for the annual per capita income. (Per capita means per person.) Estimate the per capita income in 2010. (Assume $t > 0$.)

### SOLUTION

To find a model $M$ for the annual per capita income, divide the total amount $I$ by the population $P$.

$$M = \frac{6922t + 106{,}947}{0.0063t + 1} \div (0.0343t + 4.432) \qquad \text{Divide } I \text{ by } P.$$

$$= \frac{6922t + 106{,}947}{0.0063t + 1} \cdot \frac{1}{0.0343t + 4.432} \qquad \text{Multiply by reciprocal.}$$

$$= \frac{6922t + 106{,}947}{(0.0063t + 1)(0.0343t + 4.432)} \qquad \text{Multiply.}$$

To estimate Alabama's per capita income in 2010, let $t = 10$ in the model.

$$M = \frac{6922 \cdot 10 + 106{,}947}{(0.0063 \cdot 10 + 1)(0.0343 \cdot 10 + 4.432)} \qquad \text{Substitute 10 for } t.$$

$$\approx 34{,}707 \qquad \text{Use a calculator.}$$

▶ In 2010, the per capita income in Alabama was about $34,707.

## Monitoring Progress

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**Find the quotient.**

**8.** $\frac{4x}{5x - 20} \div \frac{x^2 - 2x}{x^2 - 6x + 8}$

**9.** $\frac{2x^2 + 3x - 5}{6x} \div (2x^2 + 5x)$

# 7.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **WRITING** Describe how to multiply and divide two rational expressions.

2. **WHICH ONE DOESN'T BELONG?** Which rational expression does *not* belong with the other three? Explain your reasoning.

| | | | |
|---|---|---|---|
| $\dfrac{x-4}{x^2}$ | $\dfrac{x^2+4x-12}{x^2+6x}$ | $\dfrac{9+x}{3x^2}$ | $\dfrac{x^2-x-12}{x^2-6x}$ |

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–10, simplify the expression, if possible.** *(See Example 1.)*

3. $\dfrac{2x^2}{3x^2-4x}$

4. $\dfrac{7x^3-x^2}{2x^3}$

5. $\dfrac{x^2-3x-18}{x^2-7x+6}$

6. $\dfrac{x^2+13x+36}{x^2-7x+10}$

7. $\dfrac{x^2+11x+18}{x^3+8}$

8. $\dfrac{x^2-7x+12}{x^3-27}$

9. $\dfrac{32x^4-50}{4x^3-12x^2-5x+15}$

10. $\dfrac{3x^3-3x^2+7x-7}{27x^4-147}$

**In Exercises 11–20, find the product.** *(See Examples 2, 3, and 4.)*

11. $\dfrac{4xy^3}{x^2y} \cdot \dfrac{y}{8x}$

12. $\dfrac{48x^5y^3}{y^4} \cdot \dfrac{x^2y}{6x^3y^2}$

13. $\dfrac{x^2(x-4)}{x-3} \cdot \dfrac{(x-3)(x+6)}{x^3}$

14. $\dfrac{x^3(x+5)}{x-9} \cdot \dfrac{(x-9)(x+8)}{3x^3}$

15. $\dfrac{x^2-3x}{x-2} \cdot \dfrac{x^2+x-6}{x}$

16. $\dfrac{x^2-4x}{x-1} \cdot \dfrac{x^2+3x-4}{2x}$

17. $\dfrac{x^2+3x-4}{x^2+4x+4} \cdot \dfrac{2x^2+4x}{x^2-4x+3}$

18. $\dfrac{x^2-x-6}{4x^3} \cdot \dfrac{2x^2+2x}{x^2+5x+6}$

19. $\dfrac{x^2+5x-36}{x^2-49} \cdot (x^2-11x+28)$

20. $\dfrac{x^2-x-12}{x^2-16} \cdot (x^2+2x-8)$

21. **ERROR ANALYSIS** Describe and correct the error in simplifying the rational expression.

✗
$$\frac{x^2+\overset{2}{\cancel{16}}x+\overset{3}{\cancel{48}}}{x^2+\underset{1}{\cancel{8}}x+\underset{1}{\cancel{16}}}=\frac{x^2+2x+3}{x^2+x+1}$$

22. **ERROR ANALYSIS** Describe and correct the error in finding the product.

✗
$$\frac{x^2-25}{3-x}\cdot\frac{x-3}{x+5}=\frac{(x+5)(x-5)}{3-x}\cdot\frac{x-3}{x+5}$$
$$=\frac{\cancel{(x+5)}(x-5)\cancel{(x-3)}}{\cancel{(3-x)}\cancel{(x+5)}}$$
$$=x-5,\ x\neq 3,\ x\neq -5$$

23. **USING STRUCTURE** Which rational expression is in simplified form?

Ⓐ $\dfrac{x^2-x-6}{x^2+3x+2}$

Ⓑ $\dfrac{x^2+6x+8}{x^2+2x-3}$

Ⓒ $\dfrac{x^2-6x+9}{x^2-2x-3}$

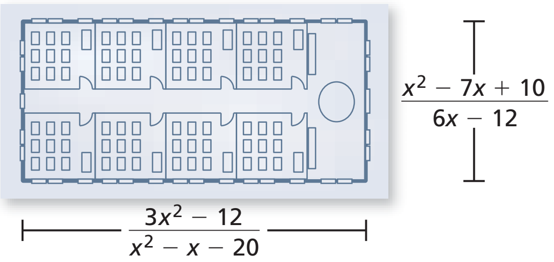
Ⓓ $\dfrac{x^2+3x-4}{x^2+x-2}$

24. **COMPARING METHODS** Find the product below by multiplying the numerators and denominators, then simplifying. Then find the product by simplifying each expression, then multiplying. Which method do you prefer? Explain.

$$\frac{4x^2y}{2x^3}\cdot\frac{12y^4}{24x^2}$$

**25. WRITING** Compare the function $f(x) = \dfrac{(3x - 7)(x + 6)}{(3x - 7)}$ to the function $g(x) = x + 6$.

**26. MODELING WITH MATHEMATICS** Write a model in terms of $x$ for the total area of the base of the building.




**In Exercises 27–34, find the quotient.** *(See Examples 5 and 6.)*

**27.** $\dfrac{32x^3y}{y^8} \div \dfrac{y^7}{8x^4}$

**28.** $\dfrac{2xyz}{x^3z^3} \div \dfrac{6y^4}{2x^2z^2}$

**29.** $\dfrac{x^2 - x - 6}{2x^4 - 6x^3} \div \dfrac{x + 2}{4x^3}$

**30.** $\dfrac{2x^2 - 12x}{x^2 - 7x + 6} \div \dfrac{2x}{3x - 3}$

**31.** $\dfrac{x^2 - x - 6}{x + 4} \div (x^2 - 6x + 9)$

**32.** $\dfrac{x^2 - 5x - 36}{x + 2} \div (x^2 - 18x + 81)$

**33.** $\dfrac{x^2 + 9x + 18}{x^2 + 6x + 8} \div \dfrac{x^2 - 3x - 18}{x^2 + 2x - 8}$

**34.** $\dfrac{x^2 - 3x - 40}{x^2 + 8x - 20} \div \dfrac{x^2 + 13x + 40}{x^2 + 12x + 20}$

**In Exercises 35 and 36, use the following information.**

*Manufacturers often package products in a way that uses the least amount of material. One measure of the efficiency of a package is the ratio of its surface area S to its volume V. The smaller the ratio, the more efficient the packaging.*

**35.** You are examining three cylindrical containers.

**a.** Write an expression for the efficiency ratio $\dfrac{S}{V}$ of a cylinder.

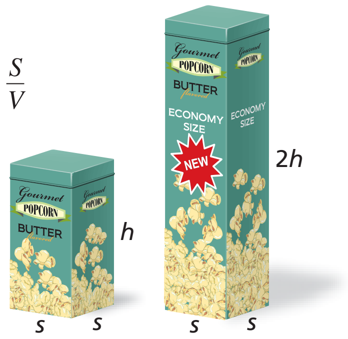
**b.** Find the efficiency ratio for each cylindrical can listed in the table. Rank the three cans according to efficiency.

| | Soup | Coffee | Paint |
|---|---|---|---|
| **Height, *h*** | 10.2 cm | 15.9 cm | 19.4 cm |
| **Radius, *r*** | 3.4 cm | 7.8 cm | 8.4 cm |

**36. PROBLEM SOLVING** A popcorn company is designing a new tin with the same square base and twice the height of the old tin.

**a.** Write an expression for the efficiency ratio $\dfrac{S}{V}$ of each tin.

**b.** Did the company make a good decision by creating the new tin? Explain.




**37. MODELING WITH MATHEMATICS** The total amount $I$ (in millions of dollars) of healthcare expenditures and the residential population $P$ (in millions) in the United States can be modeled by

$$I = \frac{171{,}000t + 1{,}361{,}000}{1 + 0.018t} \quad \text{and}$$

$$P = 2.96t + 278.649$$

where $t$ is the number of years since 2000. Find a model $M$ for the annual healthcare expenditures per resident. Estimate the annual healthcare expenditures per resident in 2010. *(See Example 7.)*

**38. MODELING WITH MATHEMATICS** The total amount $I$ (in millions of dollars) of school expenditures from prekindergarten to a college level and the enrollment $P$ (in millions) in prekindergarten through college in the United States can be modeled by

$$I = \frac{17{,}913t + 709{,}569}{1 - 0.028t} \text{ and } P = 0.5906t + 70.219$$

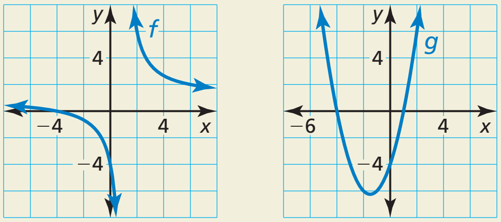
where $t$ is the number of years since 2001. Find a model $M$ for the annual education expenditures per student. Estimate the annual education expenditures per student in 2009.

**39. USING EQUATIONS** Refer to the population model $P$ in Exercise 37.

**a.** Interpret the meaning of the coefficient of $t$.

**b.** Interpret the meaning of the constant term.

**40.** **HOW DO YOU SEE IT?** Use the graphs of $f$ and $g$ to determine the excluded values of the functions $h(x) = (fg)(x)$ and $k(x) = \left(\frac{f}{g}\right)(x)$. Explain your reasoning.




**41.** **DRAWING CONCLUSIONS** Complete the table for the function $y = \frac{x+4}{x^2-16}$. Then use the *trace* feature of a graphing calculator to explain the behavior of the function at $x = -4$.

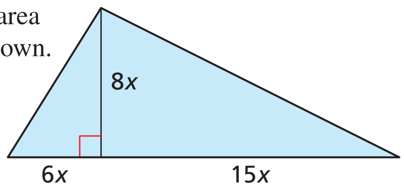
| $x$ | $y$ |
|---|---|
| −3.5 | |
| −3.8 | |
| −3.9 | |
| −4.1 | |
| −4.2 | |

**42.** **MAKING AN ARGUMENT** You and your friend are asked to state the domain of the expression below.

$$\frac{x^2+6x-27}{x^2+4x-45}$$

Your friend claims the domain is all real numbers except 5. You claim the domain is all real numbers except −9 and 5. Who is correct? Explain.

**43.** **MATHEMATICAL CONNECTIONS** Find the ratio of the perimeter to the area of the triangle shown.




**44.** **CRITICAL THINKING** Find the expression that makes the following statement true. Assume $x \neq -2$ and $x \neq 5$.

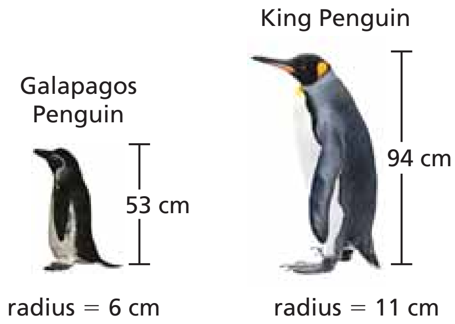
$$\frac{x-5}{x^2+2x-35} \div \frac{\boxed{\phantom{xx}}}{x^2-3x-10} = \frac{x+2}{x+7}$$

**USING STRUCTURE** **In Exercises 45 and 46, perform the indicated operations.**

**45.** $\frac{2x^2+x-15}{2x^2-11x-21} \cdot (6x+9) \div \frac{2x-5}{3x-21}$

**46.** $(x^3+8) \cdot \frac{x-2}{x^2-2x+4} \div \frac{x^2-4}{x-6}$

**47.** **REASONING** Animals that live in temperatures several degrees colder than their bodies must avoid losing heat to survive. Animals can better conserve body heat as their surface area to volume ratios decrease. Find the surface area to volume ratio of each penguin shown by using cylinders to approximate their shapes. Which penguin is better equipped to live in a colder environment? Explain your reasoning.



*Not drawn to scale*

**48.** **THOUGHT PROVOKING** Is it possible to write two radical functions whose product when graphed is a parabola and whose quotient when graphed is a hyperbola? Justify your answer.

**49.** **REASONING** Find two rational functions $f$ and $g$ that have the stated product and quotient.

$$(fg)(x) = x^2, \left(\frac{f}{g}\right)(x) = \frac{(x-1)^2}{(x+2)^2}$$

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Solve the equation. Check your solution.** *(Skills Review Handbook)*

**50.** $\frac{1}{2}x + 4 = \frac{3}{2}x + 5$ **51.** $\frac{1}{3}x - 2 = \frac{3}{4}x$ **52.** $\frac{1}{4}x - \frac{3}{5} = \frac{9}{2}x - \frac{4}{5}$ **53.** $\frac{1}{2}x + \frac{1}{3} = \frac{3}{4}x - \frac{1}{5}$

**Write the prime factorization of the number. If the number is prime, then write *prime*.** *(Skills Review Handbook)*

**54.** 42 **55.** 91 **56.** 72 **57.** 79

# 7.4 Adding and Subtracting Rational Expressions

## Essential Question

How can you determine the domain of the sum or difference of two rational expressions?

You can add and subtract rational expressions in much the same way that you add and subtract fractions.

$$\frac{x}{x+1} + \frac{2}{x+1} = \frac{x+2}{x+1}$$ Sum of rational expressions

$$\frac{1}{x} - \frac{1}{2x} = \frac{2}{2x} - \frac{1}{2x} = \frac{1}{2x}$$ Difference of rational expressions

### EXPLORATION 1 Adding and Subtracting Rational Expressions

**Work with a partner.** Find the sum or difference of the two rational expressions. Then match the sum or difference with its domain. Explain your reasoning.

| Sum or Difference | Domain |
|---|---|
| **a.** $\frac{1}{x-1} + \frac{3}{x-1} =$ | **A.** all real numbers except $-2$ |
| **b.** $\frac{1}{x-1} + \frac{1}{x} =$ | **B.** all real numbers except $-1$ and $1$ |
| **c.** $\frac{1}{x-2} + \frac{1}{2-x} =$ | **C.** all real numbers except $1$ |
| **d.** $\frac{1}{x-1} + \frac{-1}{x+1} =$ | **D.** all real numbers except $0$ |
| **e.** $\frac{x}{x+2} - \frac{x+1}{2+x} =$ | **E.** all real numbers except $-2$ and $1$ |
| **f.** $\frac{x}{x-2} - \frac{x+1}{x} =$ | **F.** all real numbers except $0$ and $1$ |
| **g.** $\frac{x}{x+2} - \frac{x}{x-1} =$ | **G.** all real numbers except $2$ |
| **h.** $\frac{x+2}{x} - \frac{x+1}{x} =$ | **H.** all real numbers except $0$ and $2$ |

**CONSTRUCTING VIABLE ARGUMENTS**

To be proficient in math, you need to justify your conclusions and communicate them to others.

### EXPLORATION 2 Writing a Sum or Difference

**Work with a partner.** Write a sum or difference of rational expressions that has the given domain. Justify your answer.

**a.** all real numbers except $-1$

**b.** all real numbers except $-1$ and $3$

**c.** all real numbers except $-1$, $0$, and $3$

## Communicate Your Answer

**3.** How can you determine the domain of the sum or difference of two rational expressions?

**4.** Your friend found a sum as follows. Describe and correct the error(s).

$$\frac{x}{x+4} + \frac{3}{x-4} = \frac{x+3}{2x}$$

# 7.4 Lesson

## Core Vocabulary

complex fraction, *p. 387*

***Previous***
rational numbers
reciprocal

## What You Will Learn

- Add or subtract rational expressions.
- Rewrite rational expressions and graph the related function.
- Simplify complex fractions.

## Adding or Subtracting Rational Expressions

As with numerical fractions, the procedure used to add (or subtract) two rational expressions depends upon whether the expressions have like or unlike denominators. To add (or subtract) rational expressions with like denominators, simply add (or subtract) their numerators. Then place the result over the common denominator.

### Core Concept

**Adding or Subtracting with Like Denominators**

Let $a$, $b$, and $c$ be expressions with $c \neq 0$.

**Addition**

$$\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$$

**Subtraction**

$$\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$$

### EXAMPLE 1 Adding or Subtracting with Like Denominators

**a.** $\dfrac{7}{4x} + \dfrac{3}{4x} = \dfrac{7 + 3}{4x} = \dfrac{10}{4x} = \dfrac{5}{2x}$ Add numerators and simplify.

**b.** $\dfrac{2x}{x + 6} - \dfrac{5}{x + 6} = \dfrac{2x - 5}{x + 6}$ Subtract numerators.

### Monitoring Progress

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**Find the sum or difference.**

**1.** $\dfrac{8}{12x} - \dfrac{5}{12x}$ **2.** $\dfrac{2}{3x^2} + \dfrac{1}{3x^2}$ **3.** $\dfrac{4x}{x - 2} - \dfrac{x}{x - 2}$ **4.** $\dfrac{2x^2}{x^2 + 1} + \dfrac{2}{x^2 + 1}$

To add (or subtract) two rational expressions with *unlike* denominators, find a common denominator. Rewrite each rational expression using the common denominator. Then add (or subtract).

### Core Concept

**Adding or Subtracting with Unlike Denominators**

Let $a$, $b$, $c$, and $d$ be expressions with $c \neq 0$ and $d \neq 0$.

**Addition**

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad + bc}{cd}$$

**Subtraction**

$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad - bc}{cd}$$

You can always find a common denominator of two rational expressions by multiplying the denominators, as shown above. However, when you use the least common denominator (LCD), which is the least common multiple (LCM) of the denominators, simplifying your answer may take fewer steps.

To find the LCM of two (or more) expressions, factor the expressions completely. The LCM is the product of the highest power of each factor that appears in any of the expressions.

### EXAMPLE 2 Finding a Least Common Multiple (LCM)

Find the least common multiple of $4x^2 - 16$ and $6x^2 - 24x + 24$.

**SOLUTION**

**Step 1** Factor each polynomial. Write numerical factors as products of primes.

$$4x^2 - 16 = 4(x^2 - 4) = (2^2)(x + 2)(x - 2)$$

$$6x^2 - 24x + 24 = 6(x^2 - 4x + 4) = (2)(3)(x - 2)^2$$

**Step 2** The LCM is the product of the highest power of each factor that appears in either polynomial.

$$\text{LCM} = (2^2)(3)(x + 2)(x - 2)^2 = 12(x + 2)(x - 2)^2$$

### EXAMPLE 3 Adding with Unlike Denominators

Find the sum $\dfrac{7}{9x^2} + \dfrac{x}{3x^2 + 3x}$.

**SOLUTION**

**Method 1** Use the definition for adding rational expressions with unlike denominators.

$$\frac{7}{9x^2} + \frac{x}{3x^2 + 3x} = \frac{7(3x^2 + 3x) + x(9x^2)}{9x^2(3x^2 + 3x)} \qquad \frac{a}{c} + \frac{b}{d} = \frac{ad + bc}{cd}$$

$$= \frac{21x^2 + 21x + 9x^3}{9x^2(3x^2 + 3x)} \qquad \text{Distributive Property}$$

$$= \frac{\cancel{3x}(3x^2 + 7x + 7)}{9x^2(x + 1)(\cancel{3x})} \qquad \text{Factor. Divide out common factors.}$$

$$= \frac{3x^2 + 7x + 7}{9x^2(x + 1)} \qquad \text{Simplify.}$$

**Method 2** Find the LCD and then add. To find the LCD, factor each denominator and write each factor to the highest power that appears in either denominator. Note that $9x^2 = 3^2x^2$ and $3x^2 + 3x = 3x(x + 1)$, so the LCD is $9x^2(x + 1)$.

$$\frac{7}{9x^2} + \frac{x}{3x^2 + 3x} = \frac{7}{9x^2} + \frac{x}{3x(x + 1)} \qquad \text{Factor second denominator.}$$

$$= \frac{7}{9x^2} \cdot \frac{x + 1}{x + 1} + \frac{x}{3x(x + 1)} \cdot \frac{3x}{3x} \qquad \text{LCD is } 9x^2(x + 1).$$

$$= \frac{7x + 7}{9x^2(x + 1)} + \frac{3x^2}{9x^2(x + 1)} \qquad \text{Multiply.}$$

$$= \frac{3x^2 + 7x + 7}{9x^2(x + 1)} \qquad \text{Add numerators.}$$

Note in Examples 1 and 3 that when adding or subtracting rational expressions, the result is a rational expression. In general, similar to rational numbers, rational expressions are closed under addition and subtraction.

## EXAMPLE 4 Subtracting with Unlike Denominators

Find the difference $\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$.

### SOLUTION

**COMMON ERROR**

When subtracting rational expressions, remember to distribute the negative sign to all the terms in the quantity that is being subtracted.

$$\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3} = \frac{x+2}{2(x-1)} - \frac{-2x-1}{(x-1)(x-3)}$$ Factor each denominator.

$$= \frac{x+2}{2(x-1)} \cdot \frac{x-3}{x-3} - \frac{-2x-1}{(x-1)(x-3)} \cdot \frac{2}{2}$$ LCD is $2(x-1)(x-3)$.

$$= \frac{x^2-x-6}{2(x-1)(x-3)} - \frac{-4x-2}{2(x-1)(x-3)}$$ Multiply.

$$= \frac{x^2-x-6-(-4x-2)}{2(x-1)(x-3)}$$ Subtract numerators.

$$= \frac{x^2+3x-4}{2(x-1)(x-3)}$$ Simplify numerator.

$$= \frac{\cancel{(x-1)}(x+4)}{2\cancel{(x-1)}(x-3)}$$ Factor numerator. Divide out common factors.

$$= \frac{x+4}{2(x-3)}, \, x \neq -1$$ Simplify.

### Monitoring Progress

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**5.** Find the least common multiple of $5x^3$ and $10x^2 - 15x$.

**Find the sum or difference.**

**6.** $\frac{3}{4x} - \frac{1}{7}$ **7.** $\frac{1}{3x^2} + \frac{x}{9x^2-12}$ **8.** $\frac{x}{x^2-x-12} + \frac{5}{12x-48}$

## Rewriting Rational Functions

Rewriting a rational expression may reveal properties of the related function and its graph. In Example 4 of Section 7.2, you used long division to rewrite a rational expression. In the next example, you will use inspection.

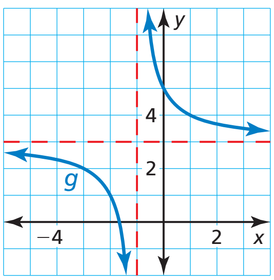
## EXAMPLE 5 Rewriting and Graphing a Rational Function

Rewrite the function $g(x) = \frac{3x+5}{x+1}$ in the form $g(x) = \frac{a}{x-h} + k$. Graph the function. Describe the graph of $g$ as a transformation of the graph of $f(x) = \frac{a}{x}$.

### SOLUTION

Rewrite by inspection:

$$\frac{3x+5}{x+1} = \frac{3x+3+2}{x+1} = \frac{3(x+1)+2}{x+1} = \frac{3\cancel{(x+1)}}{\cancel{x+1}} + \frac{2}{x+1} = 3 + \frac{2}{x+1}$$




▶ The rewritten function is $g(x) = \frac{2}{x+1} + 3$. The graph of $g$ is a translation 1 unit left and 3 units up of the graph of $f(x) = \frac{2}{x}$.

### Monitoring Progress

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**9.** Rewrite $g(x) = \frac{2x-4}{x-3}$ in the form $g(x) = \frac{a}{x-h} + k$. Graph the function. Describe the graph of $g$ as a transformation of the graph of $f(x) = \frac{a}{x}$.

## Complex Fractions

A **complex fraction** is a fraction that contains a fraction in its numerator or denominator. A complex fraction can be simplified using either of the methods below.

### Core Concept

**Simplifying Complex Fractions**

**Method 1** If necessary, simplify the numerator and denominator by writing each as a single fraction. Then divide by multiplying the numerator by the reciprocal of the denominator.

**Method 2** Multiply the numerator and the denominator by the LCD of *every* fraction in the numerator and denominator. Then simplify.

### EXAMPLE 6 Simplifying a Complex Fraction

Simplify $\dfrac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}}$.

**SOLUTION**

**Method 1**

$$\frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}} = \frac{\frac{5}{x+4}}{\frac{3x+8}{x(x+4)}}$$ Add fractions in denominator.

$$= \frac{5}{x+4} \cdot \frac{x(x+4)}{3x+8}$$ Multiply by reciprocal.

$$= \frac{5x\cancel{(x+4)}}{\cancel{(x+4)}(3x+8)}$$ Divide out common factors.

$$= \frac{5x}{3x+8},\ x \neq -4,\ x \neq 0$$ Simplify.

**Method 2** The LCD of all the fractions in the numerator and denominator is $x(x+4)$.

$$\frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}} = \frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}} \cdot \frac{x(x+4)}{x(x+4)}$$ Multiply numerator and denominator by the LCD.

$$= \frac{\frac{5}{\cancel{x+4}} \cdot x\cancel{(x+4)}}{\frac{1}{\cancel{x+4}} \cdot x\cancel{(x+4)} + \frac{2}{\cancel{x}} \cdot \cancel{x}(x+4)}$$ Divide out common factors.

$$= \frac{5x}{x+2(x+4)}$$ Simplify.

$$= \frac{5x}{3x+8},\ x \neq -4,\ x \neq 0$$ Simplify.

### Monitoring Progress

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**Simplify the complex fraction.**

**10.** $\dfrac{\frac{x}{6}-\frac{x}{3}}{\frac{x}{5}-\frac{7}{10}}$

**11.** $\dfrac{\frac{2}{x}-4}{\frac{2}{x}+3}$

**12.** $\dfrac{\frac{3}{x+5}}{\frac{2}{x-3}+\frac{1}{x+5}}$

# 7.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A fraction that contains a fraction in its numerator or denominator is called a(n) ___________.

2. **WRITING** Explain how adding and subtracting rational expressions is similar to adding and subtracting numerical fractions.

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–8, find the sum or difference.** *(See Example 1.)*

3. $\dfrac{15}{4x} + \dfrac{5}{4x}$

4. $\dfrac{x}{16x^2} - \dfrac{4}{16x^2}$

5. $\dfrac{9}{x+1} - \dfrac{2x}{x+1}$

6. $\dfrac{3x^2}{x-8} + \dfrac{6x}{x-8}$

7. $\dfrac{5x}{x+3} + \dfrac{15}{x+3}$

8. $\dfrac{4x^2}{2x-1} - \dfrac{1}{2x-1}$

**In Exercises 9–16, find the least common multiple of the expressions.** *(See Example 2.)*

9. $3x, 3(x-2)$

10. $2x^2, 4x + 12$

11. $2x, 2x(x-5)$

12. $24x^2, 8x^2 - 16x$

13. $x^2 - 25, x - 5$

14. $9x^2 - 16, 3x^2 + x - 4$

15. $x^2 + 3x - 40, x - 8$

16. $x^2 - 2x - 63, x + 7$

**ERROR ANALYSIS In Exercises 17 and 18, describe and correct the error in finding the sum.**

17. $\dfrac{2}{5x} + \dfrac{4}{x^2} = \dfrac{2+4}{5x+x^2} = \dfrac{6}{x(5+x)}$

18. $\dfrac{x}{x+2} + \dfrac{4}{x-5} = \dfrac{x+4}{(x+2)(x-5)}$

**In Exercises 19–26, find the sum or difference.** *(See Examples 3 and 4.)*

19. $\dfrac{12}{5x} - \dfrac{7}{6x}$

20. $\dfrac{8}{3x^2} + \dfrac{5}{4x}$

21. $\dfrac{3}{x+4} - \dfrac{1}{x+6}$

22. $\dfrac{9}{x-3} + \dfrac{2x}{x+1}$

23. $\dfrac{12}{x^2 + 5x - 24} + \dfrac{3}{x-3}$

24. $\dfrac{x^2 - 5}{x^2 + 5x - 14} - \dfrac{x+3}{x+7}$

25. $\dfrac{x+2}{x-4} + \dfrac{2}{x} + \dfrac{5x}{3x-1}$

26. $\dfrac{x+3}{x^2 - 25} - \dfrac{x-1}{x-5} + \dfrac{3}{x+3}$

**REASONING In Exercises 27 and 28, tell whether the statement is *always*, *sometimes*, or *never* true. Explain.**

27. The LCD of two rational expressions is the product of the denominators.

28. The LCD of two rational expressions will have a degree greater than or equal to that of the denominator with the higher degree.

29. **ANALYZING EQUATIONS** How would you begin to rewrite the function $g(x) = \dfrac{4x+1}{x+2}$ to obtain the form $g(x) = \dfrac{a}{x-h} + k$?

Ⓐ $g(x) = \dfrac{4(x+2) - 7}{x+2}$

Ⓑ $g(x) = \dfrac{4(x+2) + 1}{x+2}$

Ⓒ $g(x) = \dfrac{(x+2) + (3x-1)}{x+2}$

Ⓓ $g(x) = \dfrac{4x + 2 - 1}{x+2}$

30. **ANALYZING EQUATIONS** How would you begin to rewrite the function $g(x) = \dfrac{x}{x-5}$ to obtain the form $g(x) = \dfrac{a}{x-h} + k$?

Ⓐ $g(x) = \dfrac{x(x+5)(x-5)}{x-5}$

Ⓑ $g(x) = \dfrac{x-5+5}{x-5}$

Ⓒ $g(x) = \dfrac{x}{x-5+5}$

Ⓓ $g(x) = \dfrac{x}{x} - \dfrac{x}{5}$

**In Exercises 31–38, rewrite the function $g$ in the form $g(x) = \dfrac{a}{x-h} + k$. Graph the function. Describe the graph of $g$ as a transformation of the graph of $f(x) = \dfrac{a}{x}$.** *(See Example 5.)*

**31.** $g(x) = \dfrac{5x-7}{x-1}$

**32.** $g(x) = \dfrac{6x+4}{x+5}$

**33.** $g(x) = \dfrac{12x}{x-5}$

**34.** $g(x) = \dfrac{8x}{x+13}$

**35.** $g(x) = \dfrac{2x+3}{x}$

**36.** $g(x) = \dfrac{4x-6}{x}$

**37.** $g(x) = \dfrac{3x+11}{x-3}$

**38.** $g(x) = \dfrac{7x-9}{x+10}$

**In Exercises 39–44, simplify the complex fraction.** *(See Example 6.)*

**39.** $\dfrac{\frac{x}{3} - 6}{10 + \frac{4}{x}}$

**40.** $\dfrac{15 - \frac{2}{x}}{\frac{x}{5} + 4}$

**41.** $\dfrac{\frac{1}{2x-5} - \frac{7}{8x-20}}{\frac{x}{2x-5}}$

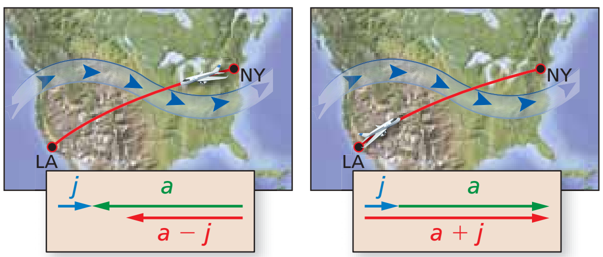
**42.** $\dfrac{\frac{16}{x-2}}{\frac{4}{x+1} + \frac{6}{x}}$

**43.** $\dfrac{\frac{1}{3x^2-3}}{\frac{5}{x+1} - \frac{x+4}{x^2-3x-4}}$

**44.** $\dfrac{\frac{3}{x-2} - \frac{6}{x^2-4}}{\frac{3}{x+2} + \frac{1}{x-2}}$

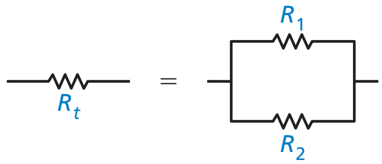
**45. PROBLEM SOLVING** The total time $T$ (in hours) needed to fly from New York to Los Angeles and back can be modeled by the equation below, where $d$ is the distance (in miles) each way, $a$ is the average airplane speed (in miles per hour), and $j$ is the average speed (in miles per hour) of the jet stream. Simplify the equation. Then find the total time it takes to fly 2468 miles when $a = 510$ miles per hour and $j = 115$ miles per hour.

$$T = \frac{d}{a-j} + \frac{d}{a+j}$$




**46. REWRITING A FORMULA** The total resistance $R_t$ of two resistors in a parallel circuit with resistances $R_1$ and $R_2$ (in ohms) is given by the equation shown. Simplify the complex fraction. Then find the total resistance when $R_1 = 2000$ ohms and $R_2 = 5600$ ohms.

$$R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$




**47. PROBLEM SOLVING** You plan a trip that involves a 40-mile bus ride and a train ride. The entire trip is 140 miles. The time (in hours) the bus travels is $y_1 = \dfrac{40}{x}$, where $x$ is the average speed (in miles per hour) of the bus. The time (in hours) the train travels is $y_2 = \dfrac{100}{x+30}$. Write and simplify a model that shows the total time $y$ of the trip.

**48. PROBLEM SOLVING** You participate in a sprint triathlon that involves swimming, bicycling, and running. The table shows the distances (in miles) and your average speed for each portion of the race.

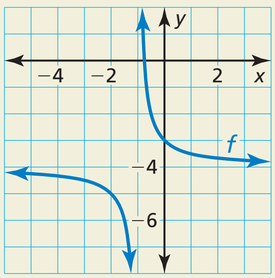
| | Distance (miles) | Speed (miles per hour) |
|---|---|---|
| **Swimming** | 0.5 | $r$ |
| **Bicycling** | 22 | $15r$ |
| **Running** | 6 | $r + 5$ |

**a.** Write a model in simplified form for the total time (in hours) it takes to complete the race.

**b.** How long does it take to complete the race if you can swim at an average speed of 2 miles per hour? Justify your answer.

**49. MAKING AN ARGUMENT** Your friend claims that the least common multiple of two numbers is always greater than each of the numbers. Is your friend correct? Justify your answer.

50. **HOW DO YOU SEE IT?** Use the graph of the function $f(x) = \frac{a}{x-h} + k$ to determine the values of $h$ and $k$.




51. **REWRITING A FORMULA** You borrow $P$ dollars to buy a car and agree to repay the loan over $t$ years at a monthly interest rate of $i$ (expressed as a decimal). Your monthly payment $M$ is given by either formula below.

$$M = \frac{Pi}{1 - \left(\frac{1}{1+i}\right)^{12t}} \quad \text{or} \quad M = \frac{Pi(1+i)^{12t}}{(1+i)^{12t} - 1}$$

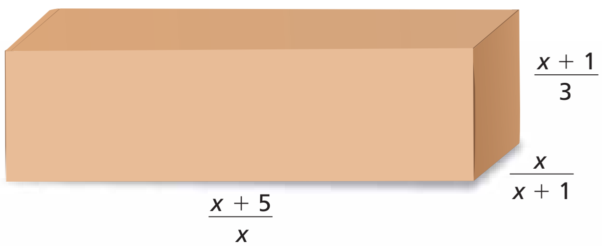
**a.** Show that the formulas are equivalent by simplifying the first formula.

**b.** Find your monthly payment when you borrow \$15,500 at a monthly interest rate of 0.5% and repay the loan over 4 years.

52. **THOUGHT PROVOKING** Is it possible to write two rational functions whose sum is a quadratic function? Justify your answer.

53. **USING TOOLS** Use technology to rewrite the function $g(x) = \frac{(97.6)(0.024) + x(0.003)}{12.2 + x}$ in the form $g(x) = \frac{a}{x-h} + k$. Describe the graph of $g$ as a transformation of the graph of $f(x) = \frac{a}{x}$.

54. **MATHEMATICAL CONNECTIONS** Find an expression for the surface area of the box.




55. **PROBLEM SOLVING** You are hired to wash the new cars at a car dealership with two other employees. You take an average of 40 minutes to wash a car ($R_1 = 1/40$ car per minute). The second employee washes a car in $x$ minutes. The third employee washes a car in $x + 10$ minutes.

**a.** Write expressions for the rates that each employee can wash a car.

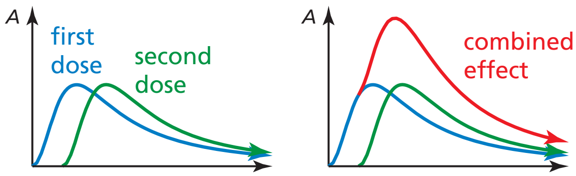
**b.** Write a single expression $R$ for the combined rate of cars washed per minute by the group.

**c.** Evaluate your expression in part (b) when the second employee washes a car in 35 minutes. How many cars per hour does this represent? Explain your reasoning.

56. **MODELING WITH MATHEMATICS** The amount $A$ (in milligrams) of aspirin in a person's bloodstream can be modeled by

$$A = \frac{391t^2 + 0.112}{0.218t^4 + 0.991t^2 + 1}$$

where $t$ is the time (in hours) after one dose is taken.




**a.** A second dose is taken 1 hour after the first dose. Write an equation to model the amount of the second dose in the bloodstream.

**b.** Write a model for the *total* amount of aspirin in the bloodstream after the second dose is taken.

57. **FINDING A PATTERN** Find the next two expressions in the pattern shown. Then simplify all five expressions. What value do the expressions approach?

$$1 + \cfrac{1}{2 + \cfrac{1}{2}},\ 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}},\ 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}},\ \ldots$$

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Solve the system by graphing.** *(Section 3.5)*

58. $y = x^2 + 6$
$y = 3x + 4$

59. $2x^2 - 3x - y = 0$
$\frac{5}{2}x - y = \frac{9}{4}$

60. $3 = y - x^2 - x$
$y = -x^2 - 3x - 5$

61. $y = (x + 2)^2 - 3$
$y = x^2 + 4x + 5$

# 7.5 Solving Rational Equations

## Essential Question

How can you solve a rational equation?

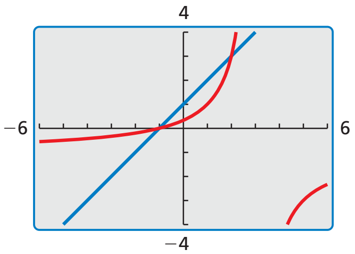
### EXPLORATION 1 Solving Rational Equations

**Work with a partner.** Match each equation with the graph of its related system of equations. Explain your reasoning. Then use the graph to solve the equation.

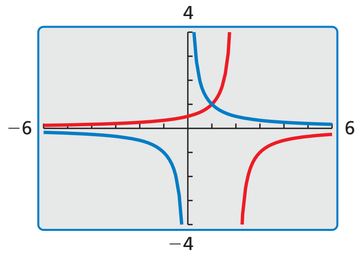
**a.** $\dfrac{2}{x-1} = 1$ **b.** $\dfrac{2}{x-2} = 2$ **c.** $\dfrac{-x-1}{x-3} = x+1$

**d.** $\dfrac{2}{x-1} = x$ **e.** $\dfrac{1}{x} = \dfrac{-1}{x-2}$ **f.** $\dfrac{1}{x} = x^2$

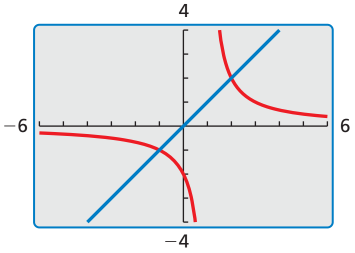
**A.**



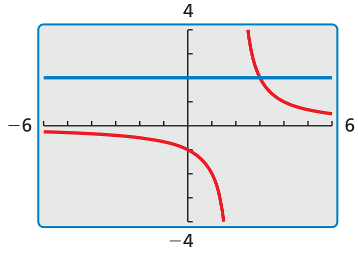

**B.**



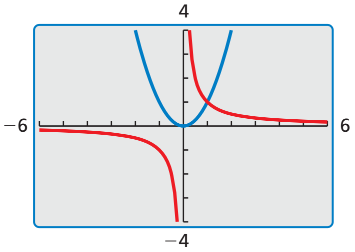

**C.**



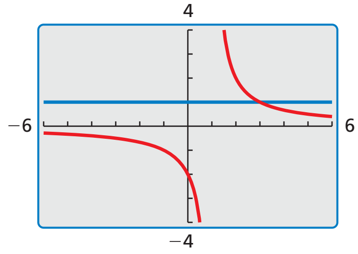

**D.**




**E.**




**F.**




**MAKING SENSE OF PROBLEMS**

To be proficient in math, you need to plan a solution pathway rather than simply jumping into a solution attempt.

### EXPLORATION 2 Solving Rational Equations

**Work with a partner.** Look back at the equations in Explorations 1(d) and 1(e). Suppose you want a more accurate way to solve the equations than using a graphical approach.

**a.** Show how you could use a *numerical approach* by creating a table. For instance, you might use a spreadsheet to solve the equations.

**b.** Show how you could use an *analytical approach*. For instance, you might use the method you used to solve proportions.

## Communicate Your Answer

**3.** How can you solve a rational equation?

**4.** Use the method in either Exploration 1 or 2 to solve each equation.

**a.** $\dfrac{x+1}{x-1} = \dfrac{x-1}{x+1}$ **b.** $\dfrac{1}{x+1} = \dfrac{1}{x^2+1}$ **c.** $\dfrac{1}{x^2-1} = \dfrac{1}{x-1}$

# 7.5 Lesson

## Core Vocabulary

cross multiplying, *p. 392*

***Previous***
proportion
extraneous solution
inverse of a function

## What You Will Learn

- Solve rational equations by cross multiplying.
- Solve rational equations by using the least common denominator.
- Use inverses of functions.

## Solving by Cross Multiplying

You can use **cross multiplying** to solve a rational equation when each side of the equation is a single rational expression.

### EXAMPLE 1 Solving a Rational Equation by Cross Multiplying

Solve $\dfrac{3}{x+1} = \dfrac{9}{4x+5}$.

**SOLUTION**

$$\frac{3}{x+1} = \frac{9}{4x+5} \qquad \text{Write original equation.}$$
$$3(4x+5) = 9(x+1) \qquad \text{Cross multiply.}$$
$$12x + 15 = 9x + 9 \qquad \text{Distributive Property}$$
$$3x + 15 = 9 \qquad \text{Subtract } 9x \text{ from each side.}$$
$$3x = -6 \qquad \text{Subtract 15 from each side.}$$
$$x = -2 \qquad \text{Divide each side by 3.}$$

▶ The solution is $x = -2$. Check this in the original equation.

**Check**

$$\frac{3}{-2+1} \stackrel{?}{=} \frac{9}{4(-2)+5}$$
$$\frac{3}{-1} \stackrel{?}{=} \frac{9}{-3}$$
$$-3 = -3 \checkmark$$

### EXAMPLE 2 Writing and Using a Rational Model

An *alloy* is formed by mixing two or more metals. Sterling silver is an alloy composed of 92.5% silver and 7.5% copper by weight. You have 15 ounces of 800 grade silver, which is 80% silver and 20% copper by weight. How much pure silver should you mix with the 800 grade silver to make sterling silver?

**SOLUTION**

$$\text{percent of copper in mixture} = \frac{\text{weight of copper in mixture}}{\text{total weight of mixture}}$$

$$\frac{7.5}{100} = \frac{(0.2)(15)}{15+x} \qquad x \text{ is the amount of silver added.}$$
$$7.5(15+x) = 100(0.2)(15) \qquad \text{Cross multiply.}$$
$$112.5 + 7.5x = 300 \qquad \text{Simplify.}$$
$$7.5x = 187.5 \qquad \text{Subtract 112.5 from each side.}$$
$$x = 25 \qquad \text{Divide each side by 7.5.}$$

▶ You should mix 25 ounces of pure silver with the 15 ounces of 800 grade silver.

## Monitoring Progress

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**Solve the equation by cross multiplying. Check your solution(s).**

**1.** $\dfrac{3}{5x} = \dfrac{2}{x-7}$ **2.** $\dfrac{-4}{x+3} = \dfrac{5}{x-3}$ **3.** $\dfrac{1}{2x+5} = \dfrac{x}{11x+8}$

## Solving by Using the Least Common Denominator

When a rational equation is not expressed as a proportion, you can solve it by multiplying each side of the equation by the least common denominator of the rational expressions.

### EXAMPLE 3 Solving Rational Equations by Using the LCD

Solve each equation.

**a.** $\frac{5}{x} + \frac{7}{4} = -\frac{9}{x}$ **b.** $1 - \frac{8}{x-5} = \frac{3}{x}$

**SOLUTION**

**a.**

$$\frac{5}{x} + \frac{7}{4} = -\frac{9}{x}$$ Write original equation.

$$4x\left(\frac{5}{x} + \frac{7}{4}\right) = 4x\left(-\frac{9}{x}\right)$$ Multiply each side by the LCD, $4x$.

$$20 + 7x = -36$$ Simplify.

$$7x = -56$$ Subtract 20 from each side.

$$x = -8$$ Divide each side by 7.

The solution is $x = -8$. Check this in the original equation.

**Check**

$$\frac{5}{-8} + \frac{7}{4} \stackrel{?}{=} -\frac{9}{-8}$$

$$-\frac{5}{8} + \frac{14}{8} \stackrel{?}{=} \frac{9}{8}$$

$$\frac{9}{8} = \frac{9}{8} \checkmark$$

**b.**

$$1 - \frac{8}{x-5} = \frac{3}{x}$$ Write original equation.

$$x(x-5)\left(1 - \frac{8}{x-5}\right) = x(x-5) \cdot \frac{3}{x}$$ Multiply each side by the LCD, $x(x-5)$.

$$x(x-5) - 8x = 3(x-5)$$ Simplify.

$$x^2 - 5x - 8x = 3x - 15$$ Distributive Property

$$x^2 - 16x + 15 = 0$$ Write in standard form.

$$(x-1)(x-15) = 0$$ Factor.

$$x = 1 \quad \text{or} \quad x = 15$$ Zero-Product Property

The solutions are $x = 1$ and $x = 15$. Check these in the original equation.

**Check**

$$1 - \frac{8}{1-5} \stackrel{?}{=} \frac{3}{1}$$ Substitute for $x$. $$1 - \frac{8}{15-5} \stackrel{?}{=} \frac{3}{15}$$

$$1 + 2 \stackrel{?}{=} 3$$ Simplify. $$1 - \frac{4}{5} \stackrel{?}{=} \frac{1}{5}$$

$$3 = 3 \checkmark \qquad \frac{1}{5} = \frac{1}{5} \checkmark$$

### Monitoring Progress

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**Solve the equation by using the LCD. Check your solution(s).**

**4.** $\frac{15}{x} + \frac{4}{5} = \frac{7}{x}$ **5.** $\frac{3x}{x+1} - \frac{5}{2x} = \frac{3}{2x}$ **6.** $\frac{4x+1}{x+1} = \frac{12}{x^2-1} + 3$

When solving a rational equation, you may obtain solutions that are extraneous. Be sure to check for extraneous solutions by checking your solutions in the *original* equation.

## EXAMPLE 4 Solving an Equation with an Extraneous Solution

Solve $\frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$.

### SOLUTION

Write each denominator in factored form. The LCD is $(x + 3)(x - 3)$.

$$\frac{6}{x-3} = \frac{8x^2}{(x+3)(x-3)} - \frac{4x}{x+3}$$

$$(x+3)(x-3) \cdot \frac{6}{x-3} = (x+3)(x-3) \cdot \frac{8x^2}{(x+3)(x-3)} - (x+3)(x-3) \cdot \frac{4x}{x+3}$$

$$6(x+3) = 8x^2 - 4x(x-3)$$

$$6x + 18 = 8x^2 - 4x^2 + 12x$$

$$0 = 4x^2 + 6x - 18$$

$$0 = 2x^2 + 3x - 9$$

$$0 = (2x-3)(x+3)$$

$$2x - 3 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -3$$

**Check**

**Check $x = \frac{3}{2}$:**

$$\frac{6}{\frac{3}{2} - 3} \stackrel{?}{=} \frac{8\left(\frac{3}{2}\right)^2}{\left(\frac{3}{2}\right)^2 - 9} - \frac{4\left(\frac{3}{2}\right)}{\frac{3}{2} + 3}$$

$$\frac{6}{-\frac{3}{2}} \stackrel{?}{=} \frac{18}{-\frac{27}{4}} - \frac{6}{\frac{9}{2}}$$

$$-4 \stackrel{?}{=} -\frac{8}{3} - \frac{4}{3}$$

$$-4 = -4 \checkmark$$

**Check $x = -3$:**

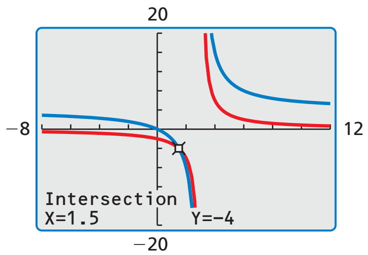
$$\frac{6}{-3 - 3} \stackrel{?}{=} \frac{8(-3)^2}{(-3)^2 - 9} - \frac{4(-3)}{-3 + 3}$$

$$\frac{6}{-6} \stackrel{?}{=} \frac{72}{0} - \frac{-12}{0} \quad ✗$$

Division by zero is undefined.

**ANOTHER WAY**

You can also graph each side of the equation and find the $x$-value where the graphs intersect.




▶ The apparent solution $x = -3$ is extraneous. So, the only solution is $x = \frac{3}{2}$.

## Monitoring Progress

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**Solve the equation. Check your solution(s).**

**7.** $\frac{9}{x-2} + \frac{6x}{x+2} = \frac{9x^2}{x^2-4}$

**8.** $\frac{7}{x-1} - 5 = \frac{6}{x^2-1}$

## Using Inverses of Functions

### EXAMPLE 5 Finding the Inverse of a Rational Function

Consider the function $f(x) = \dfrac{2}{x+3}$. Determine whether the inverse of $f$ is a function. Then find the inverse.

**SOLUTION**

Graph the function $f$. Notice that no horizontal line intersects the graph more than once. So, the inverse of $f$ is a function. Find the inverse.

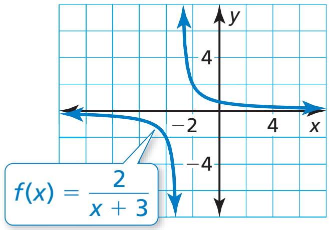
$y = \dfrac{2}{x+3}$ Set $y$ equal to $f(x)$.

$x = \dfrac{2}{y+3}$ Switch $x$ and $y$.

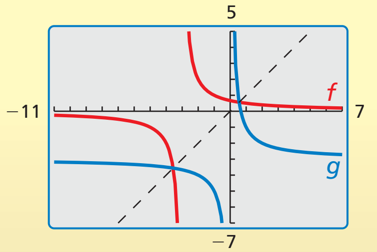
$x(y+3) = 2$ Cross multiply.

$y + 3 = \dfrac{2}{x}$ Divide each side by $x$.

$y = \dfrac{2}{x} - 3$ Subtract 3 from each side.




**Check**




▶ So, the inverse of $f$ is $g(x) = \dfrac{2}{x} - 3$.

### EXAMPLE 6 Solving a Real-Life Problem

**REMEMBER**

In part (b), the variables are meaningful. Switching them to find the inverse would create confusion. So, solve for $m$ without switching variables.

In Section 7.2 Example 5, you wrote the function $c = \dfrac{50m + 1000}{m}$, which represents the average cost $c$ (in dollars) of making $m$ models using a 3-D printer. Find how many models must be printed for the average cost per model to fall to \$90 by (a) solving an equation, and (b) using the inverse of the function.

**SOLUTION**

**a.** Substitute 90 for $c$ and solve by cross multiplying.

$$90 = \frac{50m + 1000}{m}$$

$$90m = 50m + 1000$$

$$40m = 1000$$

$$m = 25$$

**b.** Solve the equation for $m$.

$$c = \frac{50m + 1000}{m}$$

$$c = 50 + \frac{1000}{m}$$

$$c - 50 = \frac{1000}{m}$$

$$m = \frac{1000}{c - 50}$$

When $c = 90$, $m = \dfrac{1000}{90 - 50} = 25$.

▶ So, the average cost falls to \$90 per model after 25 models are printed.

## Monitoring Progress

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**9.** Consider the function $f(x) = \dfrac{1}{x} - 2$. Determine whether the inverse of $f$ is a function. Then find the inverse.

**10. WHAT IF?** How do the answers in Example 6 change when $c = \dfrac{50m + 800}{m}$?

# 7.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **WRITING** When can you solve a rational equation by cross multiplying? Explain.

2. **WRITING** A student solves the equation $\frac{4}{x-3} = \frac{x}{x-3}$ and obtains the solutions 3 and 4. Are either of these extraneous solutions? Explain.

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–10, solve the equation by cross multiplying. Check your solution(s).** *(See Example 1.)*

3. $\frac{4}{2x} = \frac{5}{x+6}$

4. $\frac{9}{3x} = \frac{4}{x+2}$

5. $\frac{6}{x-1} = \frac{9}{x+1}$

6. $\frac{8}{3x-2} = \frac{2}{x-1}$

7. $\frac{x}{2x+7} = \frac{x-5}{x-1}$

8. $\frac{-2}{x-1} = \frac{x-8}{x+1}$

9. $\frac{x^2-3}{x+2} = \frac{x-3}{2}$

10. $\frac{-1}{x-3} = \frac{x-4}{x^2-27}$

11. **USING EQUATIONS** So far in your volleyball practice, you have put into play 37 of the 44 serves you have attempted. Solve the equation $\frac{90}{100} = \frac{37+x}{44+x}$ to find the number of consecutive serves you need to put into play in order to raise your serve percentage to 90%.


12. **USING EQUATIONS** So far this baseball season, you have 12 hits out of 60 times at-bat. Solve the equation $0.360 = \frac{12+x}{60+x}$ to find the number of consecutive hits you need to raise your batting average to 0.360.

13. **MODELING WITH MATHEMATICS** Brass is an alloy composed of 55% copper and 45% zinc by weight. You have 25 ounces of copper. How many ounces of zinc do you need to make brass? *(See Example 2.)*

14. **MODELING WITH MATHEMATICS** You have 0.2 liter of an acid solution whose acid concentration is 16 moles per liter. You want to dilute the solution with water so that its acid concentration is only 12 moles per liter. Use the given model to determine how many liters of water you should add to the solution.

$$\text{Concentration of new solution} = \frac{\text{Concentration of original solution} \cdot \text{Volume of original solution}}{\text{Volume of original solution} + \text{Volume of water added}}$$

**USING STRUCTURE In Exercises 15–18, identify the LCD of the rational expressions in the equation.**

15. $\frac{x}{x+3} + \frac{1}{x} = \frac{3}{x}$

16. $\frac{5x}{x-1} - \frac{7}{x} = \frac{9}{x}$

17. $\frac{2}{x+1} + \frac{x}{x+4} = \frac{1}{2}$

18. $\frac{4}{x+9} + \frac{3x}{2x-1} = \frac{10}{3}$

**In Exercises 19–30, solve the equation by using the LCD. Check your solution(s).** *(See Examples 3 and 4.)*

19. $\frac{3}{2} + \frac{1}{x} = 2$

20. $\frac{2}{3x} + \frac{1}{6} = \frac{4}{3x}$

21. $\frac{x-3}{x-4} + 4 = \frac{3x}{x}$

22. $\frac{2}{x-3} + \frac{1}{x} = \frac{x-1}{x-3}$

23. $\frac{6x}{x+4} + 4 = \frac{2x+2}{x-1}$

24. $\frac{10}{x} + 3 = \frac{x+9}{x-4}$

25. $\frac{18}{x^2-3x} - \frac{6}{x-3} = \frac{5}{x}$

26. $\frac{10}{x^2-2x} + \frac{4}{x} = \frac{5}{x-2}$

27. $\frac{x+1}{x+6} + \frac{1}{x} = \frac{2x+1}{x+6}$

28. $\frac{x+3}{x-3} + \frac{x}{x-5} = \frac{x+5}{x-5}$

29. $\frac{5}{x} - 2 = \frac{2}{x+3}$

30. $\frac{5}{x^2+x-6} = 2 + \frac{x-3}{x-2}$

**ERROR ANALYSIS** **In Exercises 31 and 32, describe and correct the error in the first step of solving the equation.**

**31.**

$$\frac{5}{3x} + \frac{2}{x^2} = 1$$

$$3x^3 \cdot \frac{5}{3x} + 3x^3 \cdot \frac{2}{x^2} = 1$$

**32.**

$$\frac{7x+1}{2x+5} + 4 = \frac{10x-3}{3x}$$

$$(2x+5)3x \cdot \frac{7x+1}{2x+5} + 4 = \frac{10x-3}{3x} \cdot (2x+5)3x$$

**33.** **PROBLEM SOLVING** You can paint a room in 8 hours. Working together, you and your friend can paint the room in just 5 hours.

**a.** Let $t$ be the time (in hours) your friend would take to paint the room when working alone. Copy and complete the table.
(*Hint*: (Work done) = (Work rate) × (Time))

| | Work rate | Time | Work done |
|---|---|---|---|
| You | $\frac{1 \text{ room}}{8 \text{ hours}}$ | 5 hours | |
| Friend | | 5 hours | |

**b.** Explain what the sum of the expressions represents in the last column. Write and solve an equation to find how long your friend would take to paint the room when working alone.

**34.** **PROBLEM SOLVING** You can clean a park in 2 hours. Working together, you and your friend can clean the park in just 1.2 hours.

**a.** Let $t$ be the time (in hours) your friend would take to clean the park when working alone. Copy and complete the table.
(*Hint*: (Work done) = (Work rate) × (Time))

| | Work rate | Time | Work done |
|---|---|---|---|
| You | $\frac{1 \text{ park}}{2 \text{ hours}}$ | 1.2 hours | |
| Friend | | 1.2 hours | |

**b.** Explain what the sum of the expressions represents in the last column. Write and solve an equation to find how long your friend would take to clean the park when working alone.

**35.** **OPEN-ENDED** Give an example of a rational equation that you would solve using cross multiplication and one that you would solve using the LCD. Explain your reasoning.

**36.** **OPEN-ENDED** Describe a real-life situation that can be modeled by a rational equation. Justify your answer.

**In Exercises 37–44, determine whether the inverse of $f$ is a function. Then find the inverse.** *(See Example 5.)*

**37.** $f(x) = \frac{2}{x-4}$

**38.** $f(x) = \frac{7}{x+6}$

**39.** $f(x) = \frac{3}{x} - 2$

**40.** $f(x) = \frac{5}{x} - 6$

**41.** $f(x) = \frac{4}{11-2x}$

**42.** $f(x) = \frac{8}{9+5x}$

**43.** $f(x) = \frac{1}{x^2} + 4$

**44.** $f(x) = \frac{1}{x^4} - 7$

**45.** **PROBLEM SOLVING** The cost of fueling your car for 1 year can be calculated using this equation:

$$\text{Fuel cost for 1 year} = \frac{\text{Miles driven} \cdot \text{Price per gallon of fuel}}{\text{Fuel-efficiency rate}}$$

Last year you drove 9000 miles, paid \$3.24 per gallon of gasoline, and spent a total of \$1389 on gasoline. Find the fuel-efficiency rate of your car by (a) solving an equation, and (b) using the inverse of the function. *(See Example 6.)*

**46.** **PROBLEM SOLVING** The recommended percent $p$ (in decimal form) of nitrogen (by volume) in the air that a diver breathes is given by $p = \frac{105.07}{d+33}$, where $d$ is the depth (in feet) of the diver. Find the depth when the air contains 47% recommended nitrogen by (a) solving an equation, and (b) using the inverse of the function.

**USING TOOLS** **In Exercises 47–50, use a graphing calculator to solve the equation $f(x) = g(x)$.**

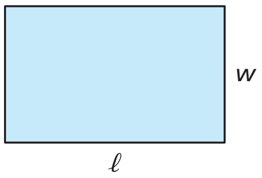
**47.** $f(x) = \frac{2}{3x}$, $g(x) = x$

**48.** $f(x) = -\frac{3}{5x}$, $g(x) = -x$

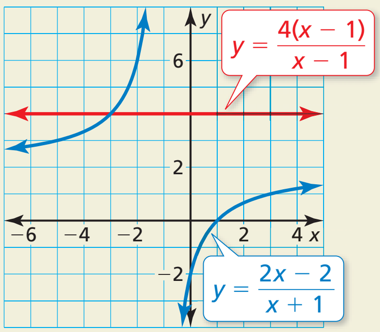
**49.** $f(x) = \frac{1}{x} + 1$, $g(x) = x^2$

**50.** $f(x) = \frac{2}{x} + 1$, $g(x) = x^2 + 1$

**51.** **MATHEMATICAL CONNECTIONS** *Golden rectangles* are rectangles for which the ratio of the width $w$ to the length $\ell$ is equal to the ratio of $\ell$ to $\ell + w$. The ratio of the length to the width for these rectangles is called the golden ratio. Find the value of the golden ratio using a rectangle with a width of 1 unit.




**52.** **HOW DO YOU SEE IT?** Use the graph to identify the solution(s) of the rational equation $\frac{4(x-1)}{x-1} = \frac{2x-2}{x+1}$. Explain your reasoning.




**USING STRUCTURE** **In Exercises 53 and 54, find the inverse of the function. (*Hint:* Try rewriting the function by using either inspection or long division.)**

**53.** $f(x) = \frac{3x+1}{x-4}$

**54.** $f(x) = \frac{4x-7}{2x+3}$

**55.** **ABSTRACT REASONING** Find the inverse of rational functions of the form $y = \frac{ax+b}{cx+d}$. Verify your answer is correct by using it to find the inverses in Exercises 53 and 54.

**56.** **THOUGHT PROVOKING** Is it possible to write a rational equation that has the following number of solutions? Justify your answers.

**a.** no solution

**b.** exactly one solution

**c.** exactly two solutions

**d.** infinitely many solutions

**57.** **CRITICAL THINKING** Let $a$ be a nonzero real number. Tell whether each statement is *always true*, *sometimes true*, or *never true*. Explain your reasoning.

**a.** For the equation $\frac{1}{x-a} = \frac{x}{x-a}$, $x = a$ is an extraneous solution.

**b.** The equation $\frac{3}{x-a} = \frac{x}{x-a}$ has exactly one solution.

**c.** The equation $\frac{1}{x-a} = \frac{2}{x+a} + \frac{2a}{x^2-a^2}$ has no solution.

**58.** **MAKING AN ARGUMENT** Your friend claims that it is not possible for a rational equation of the form $\frac{x-a}{b} = \frac{x-c}{d}$, where $b \neq 0$ and $d \neq 0$, to have extraneous solutions. Is your friend correct? Explain your reasoning.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Is the domain discrete or continuous? Explain. Graph the function using its domain.** *(Skills Review Handbook)*

**59.** The linear function $y = 0.25x$ represents the amount of money $y$ (in dollars) of $x$ quarters in your pocket. You have a maximum of eight quarters in your pocket.

**60.** A store sells broccoli for \$2 per pound. The total cost $t$ of the broccoli is a function of the number of pounds $p$ you buy.

**Evaluate the function for the given value of $x$.** *(Section 4.1)*

**61.** $f(x) = x^3 - 2x + 7$; $x = -2$

**62.** $g(x) = -2x^4 + 7x^3 + x - 2$; $x = 3$

**63.** $h(x) = -x^3 + 3x^2 + 5x$; $x = 3$

**64.** $k(x) = -2x^3 - 4x^2 + 12x - 5$; $x = -5$

# 7.3–7.5 What Did You Learn?

## Core Vocabulary

rational expression, *p. 376*
simplified form of a rational expression, *p. 376*
complex fraction, *p. 387*
cross multiplying, *p. 392*

## Core Concepts

### Section 7.3

Simplifying Rational Expressions, *p. 376*
Multiplying Rational Expressions, *p. 377*
Dividing Rational Expressions, *p. 378*

### Section 7.4

Adding or Subtracting with Like Denominators, *p. 384*
Adding or Subtracting with Unlike Denominators, *p. 384*
Simplifying Complex Fractions, *p. 387*

### Section 7.5

Solving Rational Equations by Cross Multiplying, *p. 392*
Solving Rational Equations by Using the Least Common Denominator, *p. 393*
Using Inverses of Functions, *p. 395*

## Mathematical Practices

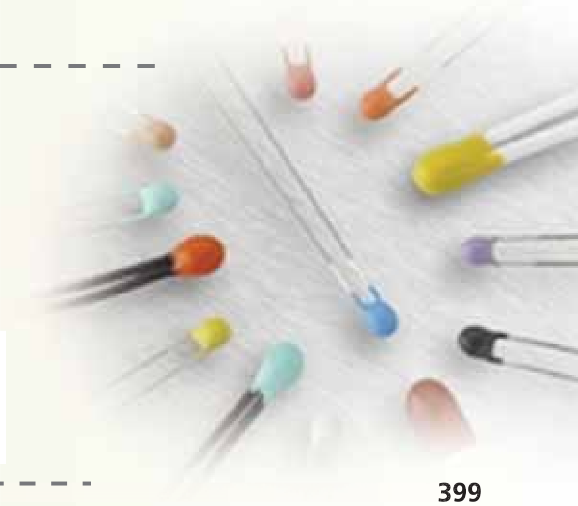
1. In Exercise 37 on page 381, what type of equation did you expect to get as your solution? Explain why this type of equation is appropriate in the context of this situation.
2. Write a simpler problem that is similar to Exercise 44 on page 382. Describe how to use the simpler problem to gain insight into the solution of the more complicated problem in Exercise 44.
3. In Exercise 57 on page 390, what conjecture did you make about the value the given expressions were approaching? What logical progression led you to determine whether your conjecture was correct?
4. Compare the methods for solving Exercise 45 on page 397. Be sure to discuss the similarities and differences between the methods as precisely as possible.

### Performance Task

## Circuit Design

A thermistor is a resistor whose resistance varies with temperature. Thermistors are an engineer's dream because they are inexpensive, small, rugged, and accurate. The one problem with thermistors is their responses to temperature are not linear. How would you design a circuit that corrects this problem?

To explore the answers to these questions and more, go to *BigIdeasMath.com*.



# 7 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

## 7.1 Inverse Variation *(pp. 359–364)*

The variables $x$ and $y$ vary inversely, and $y = 12$ when $x = 3$. Write an equation that relates $x$ and $y$. Then find $y$ when $x = -4$.

$y = \frac{a}{x}$    Write general equation for inverse variation.

$12 = \frac{a}{3}$    Substitute 12 for $y$ and 3 for $x$.

$36 = a$    Multiply each side by 3.

▶ The inverse variation equation is $y = \frac{36}{x}$. When $x = -4$, $y = \frac{36}{-4} = -9$.

**Tell whether $x$ and $y$ show *direct variation*, *inverse variation*, or *neither*.**

**1.** $xy = 5$    **2.** $5y = 6x$    **3.** $15 = \frac{x}{y}$    **4.** $y - 3 = 2x$

**5.**

| x | 7 | 11 | 15 | 20 |
|---|---|---|---|---|
| y | 35 | 55 | 75 | 100 |

**6.**

| x | 5 | 8 | 10 | 20 |
|---|---|---|---|---|
| y | 6.4 | 4 | 3.2 | 1.6 |

**The variables $x$ and $y$ vary inversely. Use the given values to write an equation relating $x$ and $y$. Then find $y$ when $x = -3$.**

**7.** $x = 1, y = 5$    **8.** $x = -4, y = -6$    **9.** $x = \frac{5}{2}, y = 18$    **10.** $x = -12, y = \frac{2}{3}$

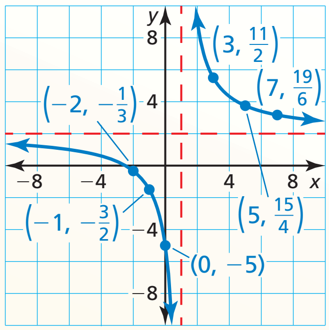
## 7.2 Graphing Rational Functions *(pp. 365–372)*

Graph $y = \frac{2x + 5}{x - 1}$. State the domain and range.

**Step 1** Draw the asymptotes. Solve $x - 1 = 0$ for $x$ to find the vertical asymptote $x = 1$. The horizontal asymptote is the line $y = \frac{a}{c} = \frac{2}{1} = 2$.

**Step 2** Plot points to the left of the vertical asymptote, such as $\left(-2, -\frac{1}{3}\right)$, $\left(-1, -\frac{3}{2}\right)$, and $(0, -5)$. Plot points to the right of the vertical asymptote, such as $\left(3, \frac{11}{2}\right)$, $\left(5, \frac{15}{4}\right)$, and $\left(7, \frac{19}{6}\right)$.

**Step 3** Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.




▶ The domain is all real numbers except 1 and the range is all real numbers except 2.

**Graph the function. State the domain and range.**

**11.** $y = \frac{4}{x - 3}$    **12.** $y = \frac{1}{x + 5} + 2$    **13.** $f(x) = \frac{3x - 2}{x - 4}$

## 7.3 Multiplying and Dividing Rational Expressions (pp. 375–382)

Find the quotient $\frac{3x + 27}{6x - 48} \div \frac{x^2 + 9x}{x^2 - 4x - 32}$.

$$\frac{3x + 27}{6x - 48} \div \frac{x^2 + 9x}{x^2 - 4x - 32} = \frac{3x + 27}{6x - 48} \cdot \frac{x^2 - 4x - 32}{x^2 + 9x}$$ Multiply by reciprocal.

$$= \frac{3(x + 9)}{6(x - 8)} \cdot \frac{(x + 4)(x - 8)}{x(x + 9)}$$ Factor.

$$= \frac{\cancel{3}\cancel{(x + 9)}(x + 4)\cancel{(x - 8)}}{2(\cancel{3})\cancel{(x - 8)}(x)\cancel{(x + 9)}}$$ Multiply. Divide out common factors.

$$= \frac{x + 4}{2x},\ x \neq 8,\ x \neq -9,\ x \neq -4$$ Simplified form

**Find the product or quotient.**

**14.** $\frac{80x^4}{y^3} \cdot \frac{xy}{5x^2}$

**15.** $\frac{x - 3}{2x - 8} \cdot \frac{6x^2 - 96}{x^2 - 9}$

**16.** $\frac{16x^2 - 8x + 1}{x^3 - 7x^2 + 12x} \div \frac{20x^2 - 5x}{15x^3}$

**17.** $\frac{x^2 - 13x + 40}{x^2 - 2x - 15} \div (x^2 - 5x - 24)$

## 7.4 Adding and Subtracting Rational Expressions (pp. 383–390)

Find the sum $\frac{x}{6x + 24} + \frac{x + 2}{x^2 + 9x + 20}$.

$$\frac{x}{6x + 24} + \frac{x + 2}{x^2 + 9x + 20} = \frac{x}{6(x + 4)} + \frac{x + 2}{(x + 4)(x + 5)}$$ Factor each denominator.

$$= \frac{x}{6(x + 4)} \cdot \frac{x + 5}{x + 5} + \frac{x + 2}{(x + 4)(x + 5)} \cdot \frac{6}{6}$$ LCD is $6(x + 4)(x + 5)$.

$$= \frac{x^2 + 5x}{6(x + 4)(x + 5)} + \frac{6x + 12}{6(x + 4)(x + 5)}$$ Multiply.

$$= \frac{x^2 + 11x + 12}{6(x + 4)(x + 5)}$$ Add numerators.

**Find the sum or difference.**

**18.** $\frac{5}{6(x + 3)} + \frac{x + 4}{2x}$

**19.** $\frac{5x}{x + 8} + \frac{4x - 9}{x^2 + 5x - 24}$

**20.** $\frac{x + 2}{x^2 + 4x + 3} - \frac{5x}{x^2 - 9}$

**Rewrite the function in the form $g(x) = \frac{a}{x - h} + k$. Graph the function. Describe the graph of $g$ as a transformation of the graph of $f(x) = \frac{a}{x}$.**

**21.** $g(x) = \frac{5x + 1}{x - 3}$

**22.** $g(x) = \frac{4x + 2}{x + 7}$

**23.** $g(x) = \frac{9x - 10}{x - 1}$

**24.** Let $f$ be the focal length of a thin camera lens, $p$ be the distance between the lens and an object being photographed, and $q$ be the distance between the lens and the film. For the photograph to be in focus, the variables should satisfy the lens equation to the right. Simplify the complex fraction.

$$f = \frac{1}{\frac{1}{p} + \frac{1}{q}}$$

## 7.5 Solving Rational Equations (pp. 391–398)

Solve $\frac{-4}{x+3} = \frac{x-1}{x+3} + \frac{x}{x-4}$.

The LCD is $(x + 3)(x - 4)$.

$$\frac{-4}{x+3} = \frac{x-1}{x+3} + \frac{x}{x-4}$$

$$(x+3)(x-4) \cdot \frac{-4}{x+3} = (x+3)(x-4) \cdot \frac{x-1}{x+3} + (x+3)(x-4) \cdot \frac{x}{x-4}$$

$$-4(x-4) = (x-1)(x-4) + x(x+3)$$

$$-4x + 16 = x^2 - 5x + 4 + x^2 + 3x$$

$$0 = 2x^2 + 2x - 12$$

$$0 = x^2 + x - 6$$

$$0 = (x+3)(x-2)$$

$$x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -3 \quad \text{or} \quad x = 2$$

**Check**

**Check $x = -3$:**

$$\frac{-4}{-3+3} \stackrel{?}{=} \frac{-3-1}{-3+3} + \frac{-3}{-3-4}$$

$$\frac{-4}{0} \stackrel{?}{=} \frac{-4}{0} + \frac{-3}{-7} \quad ✗$$

Division by zero is undefined.

**Check $x = 2$:**

$$\frac{-4}{2+3} \stackrel{?}{=} \frac{2-1}{2+3} + \frac{2}{2-4}$$

$$\frac{-4}{5} \stackrel{?}{=} \frac{1}{5} + \frac{2}{-2}$$

$$\frac{-4}{5} = \frac{-4}{5} \quad ✓$$

▶ The apparent solution $x = -3$ is extraneous. So, the only solution is $x = 2$.

**Solve the equation. Check your solution(s).**

**25.** $\frac{5}{x} = \frac{7}{x+2}$ **26.** $\frac{8(x-1)}{x^2-4} = \frac{4}{x+2}$ **27.** $\frac{2(x+7)}{x+4} - 2 = \frac{2x+20}{2x+8}$

**Determine whether the inverse of *f* is a function. Then find the inverse.**

**28.** $f(x) = \frac{3}{x+6}$ **29.** $f(x) = \frac{10}{x-7}$ **30.** $f(x) = \frac{1}{x} + 8$

**31.** At a bowling alley, shoe rentals cost \$3 and each game costs \$4. The average cost $c$ (in dollars) of bowling $n$ games is given by $c = \frac{4n+3}{n}$. Find how many games you must bowl for the average cost to fall to \$4.75 by (a) solving an equation, and (b) using the inverse of a function.

# 7 Chapter Test

**The variables $x$ and $y$ vary inversely. Use the given values to write an equation relating $x$ and $y$. Then find $y$ when $x = 4$.**

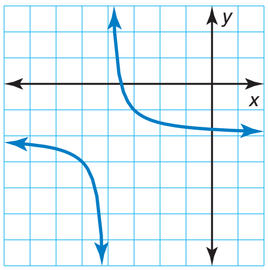
**1.** $x = 5, y = 2$

**2.** $x = -4, y = \frac{7}{2}$

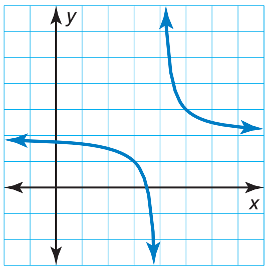
**3.** $x = \frac{3}{4}, y = \frac{5}{8}$

**The graph shows the function $y = \dfrac{1}{x - h} + k$. Determine whether the value of each constant $h$ and $k$ is *positive*, *negative*, or *zero*. Explain your reasoning.**

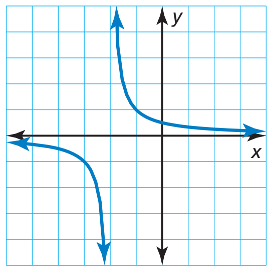
**4.**




**5.**




**6.**




**Perform the indicated operation.**

**7.** $\dfrac{3x^2y}{4x^3y^5} \div \dfrac{6y^2}{2xy^3}$

**8.** $\dfrac{3x}{x^2 + x - 12} - \dfrac{6}{x + 4}$

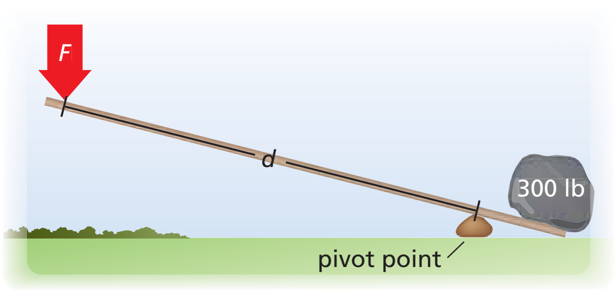
**9.** $\dfrac{x^2 - 3x - 4}{x^2 - 3x - 18} \cdot \dfrac{x - 6}{x + 1}$

**10.** $\dfrac{4}{x + 5} + \dfrac{2x}{x^2 - 25}$

**11.** Let $g(x) = \dfrac{(x + 3)(x - 2)}{x + 3}$. Simplify $g(x)$. Determine whether the graph of $f(x) = x - 2$ and the graph of $g$ are different. Explain your reasoning.

**12.** You start a small beekeeping business. Your initial costs are \$500 for equipment and bees. You estimate it will cost \$1.25 per pound to collect, clean, bottle, and label the honey. How many pounds of honey must you produce before your average cost per pound is \$1.79? Justify your answer.

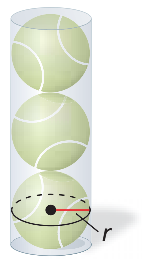
**13.** You can use a simple lever to lift a 300-pound rock. The force $F$ (in foot-pounds) needed to lift the rock is inversely related to the distance $d$ (in feet) from the pivot point of the lever. To lift the rock, you need 60 pounds of force applied to a lever with a distance of 10 feet from the pivot point. What force is needed when you increase the distance to 15 feet from the pivot point? Justify your answer.




**14.** Three tennis balls fit tightly in a can as shown.

**a.** Write an expression for the height $h$ of the can in terms of its radius $r$. Then rewrite the formula for the volume of a cylinder in terms of $r$ only.

**b.** Find the percent of the can's volume that is *not* occupied by tennis balls.



# 7 Cumulative Assessment

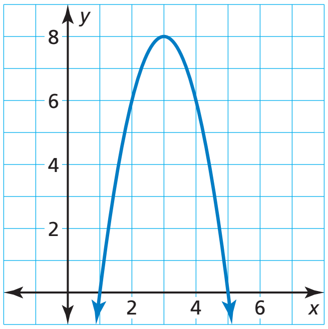
**1.** Which of the following functions are shown in the graph? Select all that apply. Justify your answers.

Ⓐ $y = -2x^2 + 12x - 10$

Ⓑ $y = x^2 - 6x + 13$

Ⓒ $y = -2(x - 3)^2 + 8$

Ⓓ $y = -(x - 1)(x - 5)$




**2.** You step onto an escalator and begin descending. After riding for 12 feet, you realize that you dropped your keys on the upper floor and walk back up the escalator to retrieve them. The total time $T$ of your trip down and up the escalator is given by

$$T = \frac{12}{s} + \frac{12}{w - s}$$

where $s$ is the speed of the escalator and $w$ is your walking speed. The trip took 9 seconds, and you walk at a speed of 6 feet per second. Find two possible speeds of the escalator.

**3.** The graph of a rational function has asymptotes that intersect at the point (4, 3). Choose the correct values to complete the equation of the function. Then graph the function.

$$y = \frac{\square x + 6}{\square x + \square}$$

| | |
|---|---|
| 12 | −3 |
| 9 | −6 |
| 3 | −12 |

**4.** The tables below give the amounts $A$ (in dollars) of money in two different bank accounts over time $t$ (in years).

| Checking Account | | | | |
|---|---|---|---|---|
| ***t*** | 1 | 2 | 3 | 4 |
| ***A*** | 5000 | 5110 | 5220 | 5330 |

| Savings Account | | | | |
|---|---|---|---|---|
| ***t*** | 1 | 2 | 3 | 4 |
| ***A*** | 5000 | 5100 | 5202 | 5306.04 |

**a.** Determine the type of function represented by the data in each table.

**b.** Provide an explanation for the type of growth of each function.

**c.** Which account has a greater value after 10 years? after 15 years? Justify your answers.

5. Order the expressions from least to greatest. Justify your answer.

| | |
|---|---|
| $5$ | $\left(\sqrt[3]{125}\right)^2$ |
| $25^{1/2}$ | $\left(\sqrt{25}\right)^2$ |
| $125^{3/2}$ | $\left(\sqrt{5}\right)^3$ |

6. A movie grosses $37 million after the first week of release. The weekly gross sales $y$ decreases by 30% each week. Write an exponential decay function that represents the weekly gross sales in week $x$. What is a reasonable domain and range in this situation? Explain your reasoning.

7. Choose the correct relationship among the variables in the table. Justify your answer by writing an equation that relates $p$, $q$, and $r$.

| $p$ | $q$ | $r$ |
|---|---|---|
| $-12$ | $20$ | $16$ |
| $3$ | $1$ | $-10$ |
| $30$ | $-82$ | $-8$ |
| $-1.5$ | $4$ | $0.5$ |

Ⓐ The variable $p$ varies directly with the difference of $q$ and $r$.

Ⓑ The variable $r$ varies inversely with the difference of $p$ and $q$.

Ⓒ The variable $q$ varies inversely with the sum of $p$ and $r$.

Ⓓ The variable $p$ varies directly with the sum of $q$ and $r$.

8. You have taken five quizzes in your history class, and your average score is 83 points. You think you can score 95 points on each remaining quiz. How many quizzes do you need to take to raise your average quiz score to 90 points? Justify your answer.