

Chapter 11

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**Chapter
11****Data Analysis and Statistics**

Dear Family,

What types of surveys have you participated in? Did you volunteer for the survey or did the surveyor contact you? The Internet has changed the way we do business with each other. Examples include buying and selling merchandise, researching contractors to hire, and restaurants to eat at. Within an online community, there is a reputation system in place such that the consumer can rate a product, service, packaging, and more. Other consumers use this information to decide whether or not to trust a store, buy a product, or make a purchase. Most places use a five-star rating system such that five is the best and one is the worst.



As a family, discuss the following questions:

- What are the benefits of the reputation system?
- What are the drawbacks of the reputation system?
- What stores have you rated?
- What products have you rated?
- Does the rating system help you when you are making a decision?
- What are some things to consider when comparing the ratings of different products or stores?
- What are some strategies business owners can do to boost their ratings?

People who leave feedback are a self-selected sample, so the ratings may be biased. In this chapter, you will learn about the different ways to conduct an experiment by choosing a sample of the population, collecting the data, and analyzing the data. From the data, you can make predictions about the population.

The next time you buy something online, remember to leave feedback for other consumers!

Capítulo
11**Análisis de datos y estadística**

Estimada familia:

¿En qué tipos de encuestas han participado? ¿Se ofrecieron a responder la encuesta o el encuestador los contactó? Internet ha cambiado la manera en que hacemos negocios con los demás. Entre los ejemplos, se incluyen comprar y vender mercadería, investigar contratistas para contratar o restaurantes donde comer. Dentro de una comunidad en línea, hay un sistema de reputación para que el comprador pueda calificar un producto, servicio, embalaje y más. Otros compradores usan esta información para decidir si confían o no en una tienda, compran un producto o hacen una compra. La mayoría de los lugares usan un sistema de calificación de cinco estrellas, donde cinco es el mejor y uno es el peor.



En familia, comenten las siguientes preguntas:

- ¿Cuáles son los beneficios de un sistema de reputación?
- ¿Cuáles son las desventajas de un sistema de reputación?
- ¿Qué tiendas ustedes han calificado?
- ¿Qué productos ustedes han calificado?
- ¿El sistema de calificación los ayuda cuando están por tomar una decisión?
- ¿Cuáles son algunas cosas a considerar cuando comparan las calificaciones de diferentes productos o tiendas?
- ¿Cuáles son algunas estrategias que pueden implementar los dueños de los negocios para aumentar sus calificaciones?

Las personas que dejan comentarios son una muestra autoseleccionada, entonces las calificaciones pueden ser tendenciosas. En este capítulo, aprenderán sobre las diferentes maneras de llevar a cabo un experimento eligiendo una muestra de población, recopilando datos y analizando los datos. A partir de los datos, pueden hacer predicciones sobre la población.

La próxima vez que compren algo en línea, recuerden dejar un comentario para otros compradores!

11.1 Start Thinking

The scores for a recent calculus test are shown in the table. Find the mean of the data set and complete the frequency table using the described intervals. Use your frequency table to create a histogram. Does your histogram appear to have a symmetric distribution?

Calculus Test Scores

54	70	78	84
56	72	79	84
60	72	79	87
62	75	80	90
68	76	83	91

Score **Frequency**

50–59	
60–69	
70–79	
80–89	
90–99	

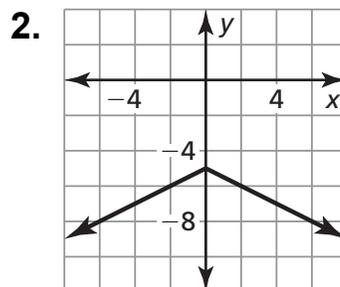
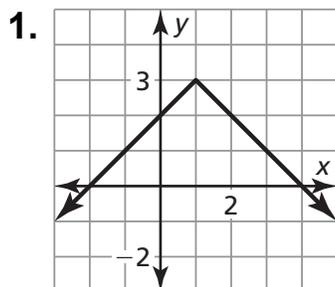
11.1 Warm Up

Find the mean and the median of the data set. Round to the nearest hundredth, when necessary.

- 18, 22, 15, 12, 25, 13, 21, 16, 22, 17
- 120, 140, 108, 115, 116, 135, 121
- 1.8, 4.5, 3.2, 6.1, 4.7, 5.1, 4.3, 5.7

11.1 Cumulative Review Warm Up

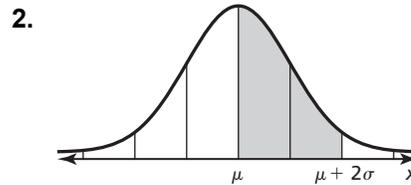
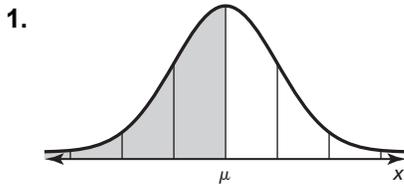
Write a function $g(x)$ whose graph represents the transformation of the graph of $f(x) = |x|$.



11.1

Practice A

In Exercises 1 and 2, give the percent of the area under the normal curve represented by the shaded region.

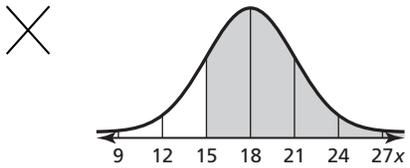


In Exercises 3–5, a normal distribution has mean μ and standard deviation σ . Find the indicated probability for a randomly selected x -value from the distribution.

3. $P(x \geq \mu + 3\sigma)$ 4. $P(x \geq \mu - 3\sigma)$ 5. $P(\mu - 2\sigma \leq x \leq \mu)$

In Exercises 6–8, a normal distribution has a mean of 28 and a standard deviation of 3. Find the probability that a randomly selected x -value from the distribution is in the given interval.

6. between 25 and 31 7. between 22 and 28 8. at least 31
9. The scores on an entrance mathematics exam are normally distributed with a mean of 76 and a standard deviation of 9.
- a. About what percent of students have scores between 67 and 94?
- b. About what percent of students have scores above 85?
10. A normal distribution has a mean of 18 and a standard deviation of 3. Describe and correct the error in finding the probability that a randomly selected x -value is in the given interval.



9 12 15 18 21 24 27 x

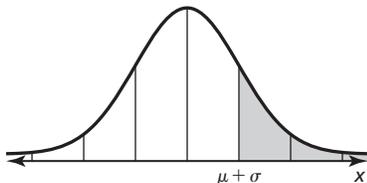
The probability that x is at least 15 is 0.8385.

11.1

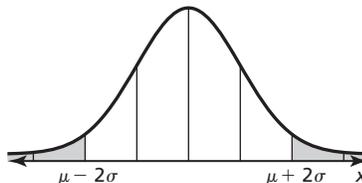
Practice B

In Exercises 1 and 2, give the percent of the area under the normal curve represented by the shaded region.

1.



2.



In Exercises 3–5, a normal distribution has mean μ and standard deviation σ .

Find the indicated probability for a randomly selected x -value from the distribution.

3. $P(x \geq \mu - 2\sigma)$

4. $P(\mu - \sigma \leq x \leq \mu + 3\sigma)$

5. $P(\mu + \sigma \leq x \leq \mu + 2\sigma)$

In Exercises 6–8, a normal distribution has a mean of 28 and a standard deviation of 3. Find the probability that a randomly selected x -value from the distribution is in the given interval.

6. between 19 and 34

7. at most 31

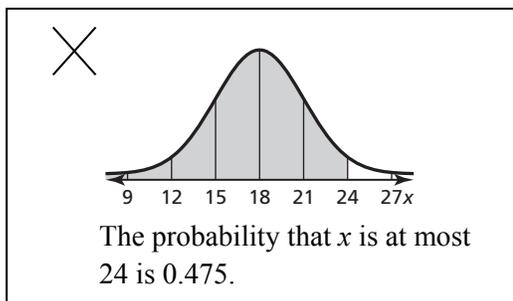
8. at least 34

9. The times a restaurant takes to prepare its "quick lunch" specials are normally distributed with a mean of 3 minutes and a standard deviation of 0.5 minute.

a. About what percent of customers have their "quick lunch" between 2 minutes and 4 minutes?

b. About what percent of customers have their "quick lunch" in fewer than 2 minutes?

10. A normal distribution has a mean of 18 and a standard deviation of 3. Describe and correct the error in finding the probability that a randomly selected x -value is in the given interval.



11.1 Enrichment and Extension

Using Normal Distributions

Pearson's Coefficient of Skewness is a measure drawn from a frequency distribution to measure the direction of variation. It is a measure of how far positively skewed (below the mean) or negatively skewed (above the mean) the majority of the data lies. It is useful when conducting a study using histograms.

Pearson first came up with the formula $P = \frac{3(\bar{X} - \text{mode})}{s}$, where \bar{X} is the mean and s is the standard deviation.

Then he also created the formula $P = \frac{3(\bar{X} - \text{median})}{s}$.

If the index is between -1 and 1 , then the distribution is approximately symmetric. If the index is less than -1 then it is skewed to the left, and if it is more than 1 , then it is skewed to the right.

Use the formula containing the median to show the skewness of the data.

- the price (in cents per pound) of bananas reported from 14 markets surveyed by the U.S. Department of Agriculture:

51	52	45	48	53	52	50
49	52	48	43	46	45	42

- the change in score (Friday–Monday) of 25 students who were given a vocabulary test on Friday and then retested when they returned to school on Monday:

6	0	-1	10	4
5	4	4	9	2
13	3	3	0	7
3	-2	4	9	9
9	6	-5	11	1

- the number of pieces of mail received at a school office for 36 days:

112	70	90	551	115	97	80	78	72
100	128	130	52	103	138	66	135	76
112	92	93	143	900	88	218	118	106
110	75	60	95	131	559	710	405	685

11.1 Puzzle Time

What is Moby Dick's Favorite Dinner?

Write the letter of each answer in the box containing the exercise number.

A normal distribution has mean μ and standard deviation σ . Find the indicated probability for a randomly selected x -value from the distribution.

1. $P(x \geq \mu + \sigma)$
2. $P(x \geq \mu - 2\sigma)$
3. $P(x \leq \mu - 2\sigma)$
4. $P(\mu - 2\sigma \leq x \leq \mu + 2\sigma)$
5. $P(x \leq \mu + 3\sigma)$

A normal distribution has a mean of 24 and a standard deviation of 3. Find the probability that a randomly selected x -value from the distribution is in the given interval.

6. between 18 and 27
7. between 21 and 33
8. between 24 and 33
9. at least 27
10. at most 15
11. not greater than 24
12. not less than 18

Answers	
S.	2.5%
A.	99.85%
F.	16%
H.	95%
S.	97.5%
N.	81.5%
S.	49.85%
I.	97.5%
I.	0.15%
H.	16%
D.	83.85%
P.	50%

1	2	3	4		5	6	7		8	9	10	11	12
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11.2 Start Thinking

When three coins are tossed, what is the theoretical probability that all three will land heads up? Conduct an experiment to verify your answer. How many times did you have to toss the three coins before you got all heads? Compare your results with your classmates and discuss how the experimental probability relates to the theoretical probability. What would you expect to be the relationship between the two probabilities as you increase the number of times you toss the coins?

11.2 Warm Up

Describe the sample space for the experiment.

1. A card is drawn from a regular deck of 52 cards.
2. A spinner that contains four colors, red, blue, yellow and green, is spun twice.
3. Each member of your class is asked if they want to participate in a mud run.
4. For one full day, patients at a clinic are asked if the wait time is acceptable.

11.2 Cumulative Review Warm Up

Solve the system of linear equations.

- | | | |
|---|---|-------------------------------------|
| 1. $2x - y = 8$
$x + 3y = 4$ | 2. $x - y = 8$
$-2x + 2y = -16$ | 3. $3x + 5y = -1$
$-2x + 3y = 7$ |
| 4. $\frac{1}{2}x - \frac{2}{3}y = \frac{5}{2}$
$\frac{1}{4}x + \frac{4}{3}y = 0$ | 5. $y = -\frac{1}{2}x + 5$
$y = \frac{7}{2}x - 11$ | 6. $y = x$
$y = -x$ |

11.2 Practice A

In Exercises 1 and 2, determine whether the data is collected from a population or a sample. Explain your reasoning.

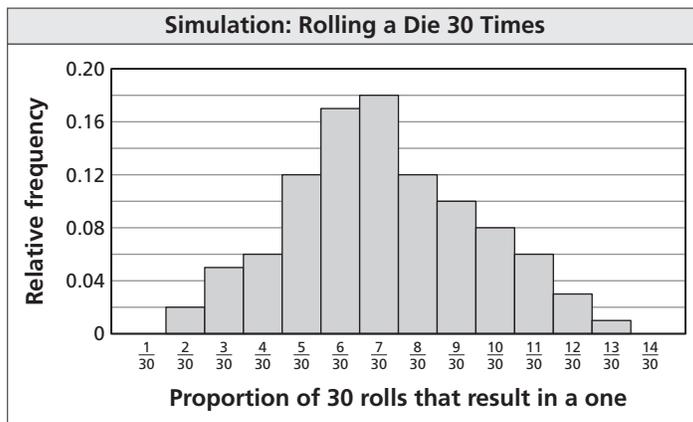
- the number of popcorn kernels in 25 bags of popcorn
- the college degrees of every employee at the hospital

In Exercises 3 and 4, identify the population and sample. Describe the sample.

- In a university, a survey of 1641 students found that 479 of them do not know the name of their college's mascot.
- In the United States, a survey of 1000 households with at least one child found that 874 of them have at least two computers.

In Exercises 5 and 6, determine whether the numerical value is a parameter or a statistic. Explain your reasoning.

- On a high school football team, 2% of the players are vegetarians.
- The average amount of the surveyed utility bills is \$176.42.
- You roll a six-sided die 5 times and get all ones. The probability of this happening is, $\left(\frac{1}{6}\right)^5 \approx 0.0001286$, so you suspect this die favors ones. The die maker claims the die does not favor ones. You simulate rolling the die 30 times by repeatedly drawing 100 samples of size 30. The histogram shows the results.



- What should you conclude when you roll the actual die 30 times and get 6 ones?
- What should you conclude when you roll the actual die 30 times and get 12 ones?

11.2 Practice B

In Exercises 1 and 2, determine whether the data is collected from a population or a sample. Explain your reasoning.

- the address of every student in the school
- a survey of 80 people who access a website

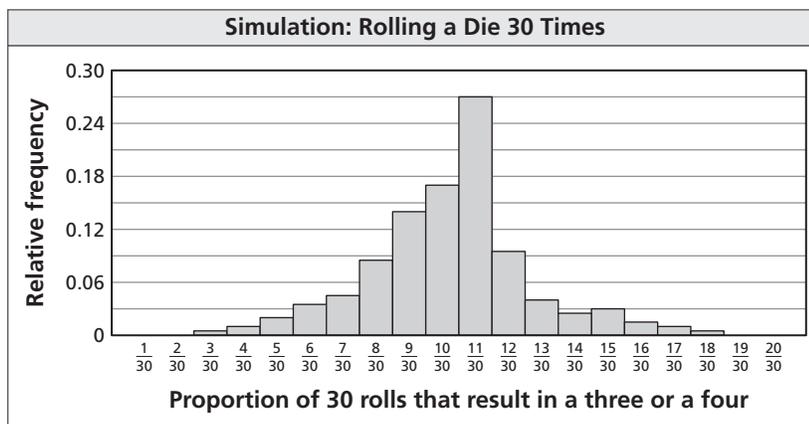
In Exercises 3 and 4, identify the population and sample. Describe the sample.

- In an office building, a survey of 648 employees found that 147 of them ride the subway to work each day.
- In Florida, a survey of 2500 homeowners found that 1145 of them have switched their homeowner's insurance policy to a different company within the last 3 years.

In Exercises 5 and 6, determine whether the numerical value is a parameter or a statistic. Explain your reasoning.

- Thirty-four percent of the surveyed hockey players first played hockey before their 10th birthday.
- Eighty-two percent of all the tickets sold were for the Saturday matinee.
- You roll a six-sided die 8 times and get either threes or fours. The probability of this happening is $\left(\frac{2}{6}\right)^8 \approx 0.0001524$, so you suspect this die favors threes and fours.

The die maker claims the die does not favor threes or fours. You simulate rolling the die 30 times by repeatedly drawing 200 samples of size 30. The histogram shows the results.



- What should you conclude when you roll the actual die 30 times and get 5 threes and fours?
- What should you conclude when you roll the actual die 30 times and get 10 threes and fours?

11.2 Enrichment and Extension

Populations, Samples, and Hypotheses

If you decide that a hypothesis is false when the hypothesis is true, this is called a Type I error. If you decide that a hypothesis is true when the hypothesis is false, this is called a Type II error. In experiments, the researcher often has to decide which error is worse.

For each of the following (a) state the Type I and Type II errors, (b) state the consequences of each error, and (c) decide which one is more serious. Explain.

Example: Researchers tested a new drug for migraine headaches against a placebo in a cross-over design. Can you conclude that the majority of the patients had better pain relief with the new drug? The hypothesis is that the new drug is not better.

Solution: a and b. Type I: Decide the new drug is better when it really is not; Consequence: Migraine patients use a pain medication that is not better than the current one on the market and do not get the necessary pain relief; Type II: Decide the new drug is not better when it really is better; Consequence: Migraine patients do not get to use a better pain medication.

c. Either could be argued, but probably Type I is more serious because people are not getting relief.

1. A recent study considered whether daily consumption of 1200 milligrams of garlic could reduce tick bites. This was tested against a placebo in a cross-over design. The hypothesis is that garlic is not effective in reducing tick bites.
2. Production managers on an assembly line must monitor the output to be sure that the level of defective products remains small. They periodically inspect a random sample of the items produced. If they find a significant increase in the proportion of items that must be rejected, they will halt the assembly process until the problem can be identified and repaired. The hypothesis is that there are not too many defects.
3. A clean air standard requires that vehicle exhaust emissions not exceed specified limits for various pollutants. Many states require that cars be tested annually to be sure they meet these standards. Suppose state regulators double-check a random sample of cars that a suspect repair shop has certified as okay. The hypothesis is that the shop is meeting car standards.
4. A statistics professor has observed that for several years about 13% of students who initially enroll in his Introductory Statistics course withdraw before the end of the semester. A salesperson suggests that he try a statistic software package that gets students more involved with computers, predicting that it will cut the dropout rate. The software is expensive, and the salesperson offers to let the professor use it for a semester to see if the dropout rate goes down significantly. The hypothesis is that the dropout rate does not decrease.

11.2 Puzzle Time

Where Do You Find Canada?

Write the letter of each answer in the box containing the exercise number.

Determine whether the data are collected from a population or a sample.

1. a survey of 400 high school sophomores at a school with 842 sophomores
2. the number of people 65 or older in the United States

Identify the population and sample.

3. In the United States, a survey of 5000 high school seniors found that 3284 of them plan on attending a 4-year university following graduation.
4. In the United States, a survey of 2500 homeowners with a monthly mortgage payment found that 1280 of them have refinanced their mortgage within the last five years.
5. In a school district, a survey of 1000 middle school students found that 894 of them participate in an extracurricular activity.
6. A survey of 5000 elementary students found that 1974 of them have cell phones.

Answers

- P.** Population: All elementary students; Sample: 5000 elementary students in the survey
- M.** Population: All homeowners with mortgage payments in the United States; Sample: 2500 homeowners in the survey
- O.** sample
- A.** Population: All high school seniors in the United States; Sample: 5000 high school seniors in the survey
- N.** population
- A.** population: All middle school students in the school district; Sample: 1000 middle school students in the survey

1	2		3		4	5	6
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11.3 Start Thinking

The purpose of a statistical study is to collect data so that an informed and reliable decision can be made. In each of the scenarios described below, determine if a random survey of 50 students from your school would provide good information for making a reliable decision.

1. The school board wants to make a decision about allocating funds for a robotics team at your school.
2. The Student Government Association at your school wants to know if families would be interested in participating in a school carnival.

11.3 Warm Up

Explain why the sample might be biased.

1. Twenty pet owners are surveyed at a veterinary clinic to determine if pets should be permitted in hotel rooms.
2. One hundred shoppers are asked if the mall hours should be extended.
3. Fifty high school students are surveyed to determine if the amount of homework should be decreased.
4. Twenty members of an athletic club were surveyed to determine how many times the average American exercises each week.

11.3 Cumulative Review Warm Up

Graph the function.

1. $y = \frac{2}{x - 5}$

2. $y = \frac{3}{x + 2} - 1$

3. $y = \frac{2x - 1}{x + 4}$

11.3 Practice A

In Exercises 1 and 2, identify the type of sample described.

1. A bank wants to know whether its drive-thru customers are satisfied with the service. Customers receive a code on their receipt that allows them to go online and fill out a survey.
2. A school wants to know whether high school teachers prefer before-school or after-school faculty meetings. Ten teachers with classrooms near the front office are surveyed.

In Exercises 3 and 4, identify the type of sample and explain why the sample is biased.

3. Every fifth customer who walks into a locally owned hardware store answers a survey that asks for opinions about lowering taxes for locally owned businesses to increase the likelihood that they can stay open for business.
4. A fitness center wants to find out whether its members would sign up for an afternoon yoga class. It surveys the first twenty members on the alphabetized membership list.

In Exercises 5 and 6, determine whether the sample is biased. Explain your reasoning.

5. Every fourth passenger who boards an airplane is asked whether they like the new method of seat assignment.
6. Your teacher is choosing 4 students to participate in a pizza-eating contest. The names of all 30 students are put in a hat and four names are randomly drawn from the hat.

In Exercises 7 and 8, explain why the survey question may be biased or otherwise introduce bias into the survey. Then describe a way to correct the flaw.

7. A guidance counselor asks high school students, "Do you miss school often?"
8. "The fish being caught in our lake are found to have lesions, which could be toxic when eaten. Do you think the city should address this health problem?"

11.3 Practice B

In Exercises 1 and 2, identify the type of sample described.

1. An amphitheater wants to know if Saturday morning concerts would be attended. The amphitheater is divided into 30 different seating areas. It surveys all the members in five randomly selected seating areas.
2. The manager of an orange juice plant wants to test the quality of the plant's new extra-pulp orange juice. She does a taste test of every twentieth container of extra-pulp orange juice.

In Exercises 3 and 4, identify the type of sample and explain why the sample is biased.

3. A town council wants to know whether residents support lower taxes for those who own waterfront property. Thirty-four owners of waterfront property are surveyed.
4. A local radio station is considering switching from country music to classical music. Randomly throughout the day it announces the phone number and website where listeners can go to place their vote for either country music or classical music.

In Exercises 5 and 6, determine whether the sample is biased. Explain your reasoning.

5. A local senator is proposing a bill to lower the limit on the number of lobster that can be caught each day. Participants in a local fishing tournament are asked to complete a survey.
6. A triathlon organizer wants to know if a Saturday morning triathlon is preferred over a Sunday morning triathlon. The organizer randomly surveys five triathletes in one of the age groups.

In Exercises 7 and 8, explain why the survey question may be biased or otherwise introduce bias into the survey. Then describe a way to correct the flaw.

7. "Our plant has not been making a profit. Do you agree that the company should close this plant?"
8. "Do you agree that the mayor is receiving a salary that is far beyond what is reasonable?"

11.3 Enrichment and Extension

Collecting Data

1. A researcher is considering three methods of evaluating two different types of fertilizer for corn. Classify each method as a survey, an experiment, or an observational study. Then identify which method would be most reliable.

Method 1	Method 2	Method 3
Randomly divide the rows of corn in the same test field into two groups. Put fertilizer on one group of rows, and record the yield for those rows and for the rows without the fertilizer. Draw conclusions based on your findings.	Choose some of the rows of corn at random from a field. Determine which type of fertilizer has been applied in the past, and what the yield is for each row. Extrapolate the findings to the whole field.	Monitor the yield of all rows of corn in various test fields being fertilized with a product chosen by the farmer. Record the yield for those rows.

2. A researcher is considering three methods of evaluating the expected charge-length of two new batteries that can be used in electric cars. Classify each method as a survey, an experiment, or an observational study. Then identify which method would be most reliable.

Method 1	Method 2	Method 3
Randomly divide some electric cars into two groups. Have each group use a different battery, and record how long the batteries last in each group before they need to be recharged. Confirm or reject your hypothesis.	Monitor the batteries in a large number of electric cars. Record how long the batteries last before they need to be recharged.	Choose a group of owners of electric cars at random. Ask which types of batteries they have used in the past, and how quickly the batteries need to be recharged. Estimate parameters for the entire population based on your data.

11.3 Puzzle Time

Why Did The Watch Go On Vacation?

Write the letter of each answer in the box containing the exercise number.

Identify the type of sample described.

1. You list all the counties in Pennsylvania and choose every third county.
2. You distribute questionnaires in the cafeteria and use only the questionnaires that are returned.
3. You ask all of the players on your baseball team.
4. You randomly select four students from each table in the cafeteria.
5. You divide all the students at a university into commuting students and campus-dwelling students, and then you randomly select all the students from the commuting group.

Identify the method of data collection each situation describes.

6. A student researcher records whether people at a restaurant leave a tip for their server.
7. A scientist provides natural light to 50 plants and fluorescent light to 50 plants in a study.
8. A teacher asks the students in her fifth-period class how many siblings they have in their immediate family so that she can draw conclusions about all her students.

Answers

N. experiment

I. observational

U. convenience

T. systematic

O. self-selected

N. stratified

W. cluster

D. survey

1	2		3	4	5	6	7	8
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11.4 Start Thinking

Discuss how you might determine if a spinner or a coin are “fair” (equally likely to come up with each possibility).

11.4 Warm Up

Identify the method of data collection the situation describes.

1. A researcher places 20 plants in natural sunlight and 20 plants in artificial light. The researcher then compares the growth of the plants after 10 weeks and determines which light is better.
2. A researcher records whether people at a gym clean the machines after they use them.

11.4 Cumulative Review Warm Up

Simplify the expression.

1. $\frac{48^{1/3}}{16^{1/3}}$

2. $(3^{1/2} \cdot 6^{1/2})^4$

3. $\sqrt{27} \sqrt[3]{27}$

4. $\frac{\sqrt{12}}{\sqrt[3]{12}}$

5. $\frac{\sqrt[5]{64} \sqrt[5]{4}}{\sqrt[5]{8}}$

6. $\frac{25^{3/2} 25^{-1}}{5^{1/2}}$

11.4**Practice A**

In Exercises 1 and 2, determine whether the study is a randomized comparative experiment. If it is, describe the treatment, the treatment group, and the control group. If it is not, explain why not and discuss whether the conclusions drawn from the study are valid.

1. To test the effect of using a computer for testing, 300 students are randomly divided into two groups. One group is tested using paper and pencil, and one group is tested using the computer. After analyzing the test results, it was found that the average test scores in both groups were not significantly different.
2. At a health fair, people can choose to enroll in a six-month healthy eating plan. Sixty people who chose the "eat six vegetables a day" plan were monitored for 6 months, as were 60 people who chose the "drink 12 glasses of water a day" plan. At the end of the 6 months, people who chose the "drink 12 glasses of water a day" plan had 20% lower cholesterol than people in the other group.

In Exercises 3 and 4, explain whether the research topic is best investigated through an experiment or an observational study. Then describe the design of the experiment or observational study.

3. A cycling team wants to know whether incorporating yoga into the workout routine improves racing times.
4. A researcher wants to compare the effects of a new experimental cancer drug with a cancer drug that has been used for at least 10 years.
5. A researcher wants to test whether stretching after exercising decreases the number of injuries due to muscle damage. Identify a potential problem, if any, with each experimental design. Then describe how you can improve it.
 - a. The researcher selects 400 people who exercise on a regular basis. The people are divided into two groups based on age. Within each age group, the people are randomly assigned to stretch after exercising or to not stretch. The people's occurrence of injuries due to muscle damage is monitored. The stretching after exercise significantly decreases the number of injuries due to muscle damage.
 - b. The researcher randomly selects 150 people who exercise on a regular basis. Half the people stretch after exercising, and the number of their injuries due to muscle damage is monitored. The other half do not stretch after exercising, and the number of their injuries due to muscle damage is monitored. The number of injuries due to muscle damage significantly decreases for those who stretch after exercising.

11.4 Practice B

In Exercises 1 and 2, determine whether the study is a randomized comparative experiment. If it is, describe the treatment, the treatment group, and the control group. If it is not, explain why not and discuss whether the conclusions drawn from the study are valid.

1. A pool cleaning service is offering a new chlorine solution to its customers. Of 90 customers, the 45 customers who chose to switch to the new chlorine solution were monitored for a year, as were the 45 customers who did not switch. At the end of the year, the customers who switched to the new chlorine solution were 30% more satisfied with the condition of their pools while those who did not switch experienced no significant change.
2. A recycling company is testing the use of recyclables containers rather than recyclables bins, in the hopes of increasing the amount of recycling. It randomly divided 150 customers into two groups. One group received the new recyclables containers, and the other group continued using their recyclables bins. After 6 months, the customers with the new recyclables containers recycled 25% more pounds of recyclables than customers with the recyclables bins.

In Exercises 3 and 4, explain whether the research topic is best investigated through an experiment or an observational study. Then describe the design of the experiment or observational study.

3. An organization wants to know whether donating 20% of one's income to charities affects one's satisfaction with his or her job.
4. A rancher wants to know whether a new feed affects the quality of the milk produced by cows.
5. A researcher wants to test whether drinking diet soda increases sugar cravings. Identify a potential problem, if any, with each experimental design. Then describe how you can improve it.
 - a. The researcher randomly selects 200 people. Half of the people drink diet soda, and the occurrence of sugar cravings is monitored. The other half of the people do not drink diet soda, and their occurrence of sugar cravings is monitored. The occurrence of sugar cravings for the people who drink diet soda is significantly higher than the occurrence of cravings for those who do not.
 - b. The researcher selects 300 people. The people are divided into two groups based on exercise habits. Within each group, the people are randomly assigned to drink diet soda or to not drink diet soda. The people's sugar cravings are monitored. There is no significant difference in the occurrence of sugar cravings between the two groups.

11.4 Enrichment and Extension

Experimental Design

The *response variable* is the variable that is being measured after the treatment is applied in an experiment. A *blind or blinded experiment* is a test or experiment in which information about the test that might lead to bias in the results is concealed from the tester, the subject, or both until after the test. Bias may be intentional or unconscious. If both tester and subject are blinded, the trial is a *double-blind trial*.

In Exercises 1–4, (a) find the treatment, (b) find the response variable, (c) find the number of treatment groups, and (d) decide if the experiment could be blind or double blind. Explain.

1. A mathematics education researcher was interested in determining the effects of class size (small, medium, and large) and the use of a traditional statistics textbook versus a new pilot textbook. The researcher conducted her own experiment, assigning each combination of class size and type of textbook to two classes at each of five chosen schools. The average final grade for each class was then recorded at the end of the year.
2. A botanist was interested in determining the effects of scheduled watering (three days a week or daily) and the use of fertilizer (traditional, and organic) in hopes of increasing the heat rating of jalapeño peppers. The botanist conducted his own experiment, assigning each combination of watering schedule and type of fertilizer to plots that had similar soil and full sun. The average final heat rating for the plot was then recorded at the end of the growing season.
3. A new dog food, specially designed for dogs with kidney problems, has been developed. A veterinarian wants to test this new food against another dog food currently on the market to see if it improves a dog's health. Thirty dogs with kidney problems were recruited to participate in the study. They were fed either the "new" or "old" food for 6 months and the improvement in kidney health was rated.
4. Does talking while walking slow you down? In a study reported in the journal *Physical Therapy*, the cadence was measured for subjects who were walking (using no device, a standard walker, or a rolling walker) and who were required (or not required) to respond to a signal while walking.
5. Researchers have developed a new insulin inhaler to replace daily insulin shots needed by patients with diabetes. Design an experiment to test the effects of this new insulin treatment on a volunteer group of 100 diabetes patients.

 **11.4 Puzzle Time****What Kind Of Hair Does The Ocean Have?**

Write the letter of each answer in the box containing the exercise number.

Identify a potential problem with the experimental design.

1. A school district's administrators want to identify whether their current after-school tutoring program is worth implementing for the following school year. A student participant from the tutoring program is given a benchmark test. A student who does not attend the program is given the same benchmark test. After grading each student's benchmark test, it is determined that the student who attends the tutoring program did not perform better than the student who does not participate in the tutoring program, so the tutoring program should not be implemented the following year.
2. The local humane society wants to test the effectiveness of a name-brand flea control medicine. The subjects are divided into a group for cats and a group for dogs, with both receiving the same medicine in proper dosages. After one week, a significantly larger number of cats have no fleas.

Explain whether the research topic is best investigated through an experiment or an observational study.

3. The manager of a grocery store produce department wants to know which type of apple more customers prefer.
4. A pet storeowner wants to know whether distilled water or natural spring water leads to longer life in the fish he sells.

Answers

- V. observational study
- W. sample size is not large enough
- Y. experiment
- A. groups are not similar

1	2	3	4
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11.5 Start Thinking

You conduct a sample survey to determine what percent of high school students in your town have a part-time job. Based on the students that you know, you predict that 25% of high school students have a part-time job. You survey 40 randomly chosen students and find that 14 of them have a part-time job. Based on your sample survey, what percent of the students have a part-time job? Do you think that you should change your prediction based on your sample survey? Explain your reasoning.

11.5 Warm Up

Sample surveys are conducted to determine the monthly cell phone bill and the number of sick or personal days taken in a calendar year. Calculate the mean, \bar{x} , of the data shown in the table. Round your answer to the nearest tenth, when necessary.

1. Monthly cell phone bill

148	210	160	190	124
165	189	194	280	215
210	160	115	220	240
228	140	169	187	218
240	215	192	115	122

2. Sick or personal days taken annually

2	4	1	0	8
3	12	6	7	5
7	4	15	4	2
1	2	9	5	7
0	3	6	5	2

11.5 Cumulative Review Warm Up

Use the Binomial Theorem to write the binomial expansion.

1. $(x - 3)^5$

2. $(2x + 5)^4$

3. $(1 - 2x)^7$

4. $(x + y^2)^4$

5. $(2a - 3b)^6$

6. $(x^3 - y^4)^8$

11.5 Practice A

1. The numbers of words per text message in a random sample of 30 text messages are shown in the table.

Number of Words Per Text Message									
3	10	9	12	21	17	4	6	18	2
10	3	9	12	5	24	19	4	1	7
5	11	14	6	2	4	14	6	9	12

- a. Estimate the population mean μ .
- b. Estimate the population proportion ρ of text messages that contain more than 12 words.
- c. Estimate the population proportion ρ of text messages that contain less than five words.
2. A survey asks a random sample of U.S. college students how many hours they spend surfing the Internet each day. The survey reveals that the sample mean is 2.7 hours per day. How confident are you that the average time spent surfing the Internet each day of all U. S. college students is exactly 2.7 hours per day? Explain your reasoning.
3. The number of households with pets is increasing. A national polling company claims that 43% of U.S. households have at least one pet. You survey a random sample of 50 households.
- a. What can you conclude about the accuracy of the claim that the population proportion is 0.43 when 29 households have at least one pet?
- b. What can you conclude about the accuracy of the claim that the population proportion is 0.43 when 22 households have at least one pet?
- c. Assume that the true proportion is 0.43. Estimate the variation among sample proportions for samples of size 50.

11.5 Practice B

1. The numbers of bait fish caught in a random sample of 40 cast net throws are shown in the table.

Number of Bait Fish Per Cast Net Throw									
4	8	6	15	1	6	8	0	1	14
9	6	8	7	3	10	4	11	2	4
15	2	0	5	1	2	7	5	6	6
9	11	8	9	5	1	4	7	2	1

- a. Estimate the population mean μ .
- b. Estimate the population proportion ρ of cast net throws that produce at least eight bait fish.
- c. Estimate the population proportion ρ of cast net throws that produce fewer than three bait fish.
2. A survey asks a random sample of U.S. voters how many times they have gone to the polls unknowledgeable about who they are voting for. The survey reveals that the sample mean is 5.8 times. How confident are you that the average number of times all U. S. voters have gone to the polls unknowledgeable is exactly 5.8 times? Explain your reasoning.
3. A national polling company claims that 45% of U.S. drivers do not adhere to the speed limits in construction areas. You survey a random sample of 50 households.
- a. What can you conclude about the accuracy of the claim that the population proportion is 0.45 when 15 drivers do not adhere to the speed limits in construction areas?
- b. What can you conclude about the accuracy of the claim that the population proportion is 0.45 when 23 drivers do not adhere to the speed limits in construction areas?
- c. Assume that the true proportion is 0.45. Estimate the variation among sample proportions for samples of size 50.

11.5 Enrichment and Extension

Making Inferences from Sample Surveys

Suppose you want to determine if you should start a new club at your school. Fill in the blanks to help you design an unbiased survey.

√	Characteristic
	1. Write the survey question.
	2. Is the survey question clear? Explain.
	3. Describe the population you will sample.
	4. Describe the method you will use to sample the population.
	5. Is your sample biased? Explain how you know.
	6. Is the sample representative of the entire population? Explain.
	7. Do a trial run of the survey with several students in your class. Re-evaluate your survey question. Does the question need modifying? Explain.
	8. Conduct the survey. Find the margin of error of your sample to help you predict the opinion of the entire school. Based on the results of the survey, make a prediction about the opinion of the entire population of the school. Did your results become more accurate the more people you surveyed?

11.5 Puzzle Time

What Do You Call A Bird That's Been Eaten By A Cat?

Write the letter of each answer in the box containing the exercise number.

1. The table shows the daily number of lunch-hour customers to a local fish market for 16 days. Estimate the population mean μ .

Number of Customers			
15	16	27	8
12	21	26	24
18	32	38	34
24	14	42	25

2. The table shows the total number of free throws made by a high school basketball team for eight consecutive games. Estimate the population mean μ .

Total Number of Free Throws Made			
24	19	7	14
13	11	12	4

Answers

L. ± 0.06

W. ± 0.1

O. ± 0.04

W. ± 0.08

A. ± 0.02

A. 23.5

L. ± 0.18

S. 13.25

Find the margin of error for a survey that has the given sample size. Round your answer to the nearest tenth of a percent.

3. 100 4. 2400 5. 32
6. 289 7. 750 8. 170

1		2	3	4	5	6	7	8
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11.6 Start Thinking

A marketing expert is consulted to help a company increase sales of their new sports drink. The expert advises them to consider repackaging the sports drink by changing the size of the bottle as well as the label. Because of the cost of such changes, the company wants to test the marketing expert's hypothesis before making a large investment in the change. How could the hypothesis be tested?

11.6 Warm Up

The table shows results of two different randomized comparative experiments. Find the mean value of the control group, the mean value of the treatment group, and the difference between the two means.

1.

Height (inches)

Control Group	14	15.2	14.2	15	15.5	15	14.8	15.5	14.1	14.6
Treatment Group	14.3	15.3	14.8	15	15.4	15.4	14.8	15.8	14.5	14.9

2.

Weight (pounds)

Control Group	58	60	61	65	57	55	62	61	60	57
Treatment Group	51	58	55	61	50	55	60	58	55	51

11.6 Cumulative Review Warm Up

Simplify the expression.

1. $\sin x \csc x$

2. $\frac{1 - \cos^2(-x)}{\cos^2 x}$

3. $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x}$

4. $\frac{\cos\left(\frac{\pi}{2} - x\right)}{\tan x}$

5. $\cos x(1 + \tan^2 x)$

6. $\frac{\sin x \cos x - \cos\left(\frac{\pi}{2} - x\right)}{\sin x}$

11.6 Practice A

1. A randomized comparative experiment tests whether pet therapy affects the mood scores of hospital patients. The mood scores range from 1 to 10, with scores close to 10 indicating a good mood. The control group has eight patients and the treatment group, which receives the pet therapy, has eight patients. The table shows the results.

	Mood Score							
Control Group	1	4	2	3	8	5	4	2
Treatment Group	3	9	4	7	3	6	2	9

- Find the mean score of the control group.
- Find the mean score of the treatment group.
- Find the experimental difference of the means.
- Display the data in a double dot plot.
- What can you conclude?
- Resample the data using a simulation.
- Find the mean score of the new control group.
- Find the mean score of the new treatment group.
- Find the experimental difference of the new means.

11.6 Practice B

1. A randomized comparative experiment tests whether photo therapy affects the size of the area of skin irritations. The control group has eight patients and the treatment group, which receives the photo therapy, has eight patients. The table shows the results.

	Area of Skin Irritation (square centimeters)							
Control Group	4.2	1.3	5.1	3.4	2.7	4.1	3.6	2.4
Treatment Group	1.2	2.5	4.1	3.7	2.1	1.5	2.4	1.9

- Find the mean area of the control group.
- Find the mean area of the treatment group.
- Find the experimental difference of the means.
- Display the data in a double dot plot.
- What can you conclude?
- Resample the data using a simulation.
- Find the mean score of the new control group.
- Find the mean score of the new treatment group.
- Find the experimental difference of the new means.

11.6 Enrichment and Extension

Making Inferences from Experiments

In statistics, a confidence interval (CI) is a type of interval estimate of a population parameter and is used to indicate the reliability of an estimate. It is an observed interval (it is calculated from observations), in principle different from sample to sample, that frequently includes the parameter of interest if the experiment is repeated. The CI formed for two sample means (which was studied in Section 11.6) looks at whether or not zero is located in the interval. If zero is located in the interval, then there is insufficient evidence that the means vary. If zero is not located in the interval, then there is sufficient evidence to suggest a difference between the means.

Example: An experimenter constructed the appropriate CI on batteries ($\mu_{\text{generic}} - \mu_{\text{brand name}}$) and found 95% of the time the interval 2.1 minutes to 35.1 minutes captures the mean amount of time by which generic batteries outlast brand-name batteries. What can the experimenter conclude from the experiment and the calculations?

Solution: Because zero is not in the interval, there is enough evidence that generic batteries do last longer than brand-name batteries. If it is cheaper and easy to get generic batteries, then there is no reason not to use them over brand-name batteries.

Decide the conclusion of the experiment based on the given confidence interval.

1. An experimenter was curious about the fat content (in grams) of two kinds of hotdogs: meat (usually a mixture of pork, turkey, and chicken) and all beef. The CI on hotdogs ($\mu_{\text{meat}} - \mu_{\text{beef}}$) found 95% of the time the interval -6.5 grams to -1.4 grams captures the mean amount of fat content between meat and beef hotdogs.
2. A man commuting to work finds two routes he could take to work. The man decides to experiment. Each day he flips a coin to determine which way to go, driving each route 20 days. He calculates the appropriate confidence interval ($\mu_{\text{route A}} - \mu_{\text{route B}}$) and finds the interval to be -1.4 minutes to 4.65 minutes.
3. An experimenter examined top-loading and front-loading washing machines, testing several different brands of each type. One of the variables was “cycle time”, the number of minutes it took each machine to wash a load of clothes. Among the machines rated good to excellent, the corresponding CI for the difference in mean cycle time ($\mu_{\text{top}} - \mu_{\text{front}}$) is (-40 minutes, -22 minutes).
4. A father decided to see if children’s cereal has a higher sugar content (as percentage of weight) than adults’ cereal. He constructed the corresponding CI for the difference in sugar content ($\mu_{\text{children’s}} - \mu_{\text{adult’s}}$) and found the interval to be 32.5% to 40.8%.

11.6 Puzzle Time

How Do You Get An Astronaut’s Baby To Fall Asleep?

Write the letter of each answer in the box containing the exercise number.

Complete the exercises.

A randomized comparative experiment tests whether time spent outdoors affects the calculus exam scores of college students during finals week. The control group has five students and the treatment group, which receives time outdoors, has five students. The table shows the results.

	Calculus Exam Score				
Control Group	80	64	95	92	71
Treatment Group	83	98	88	79	94

1. Find the mean score of the control group.
2. Find the mean score of the treatment group.
3. Find the experimental difference of the means.

A randomized comparative experiment tests whether tire tread thickness affects the gas mileage of a vehicle. The control group, which receives tires that are 1 year old, has 10 vehicles and the treatment group, which receives brand new tires, has 10 vehicles. All vehicles are identical except for their tires. The table shows the results.

	Vehicle Gas Mileage (mi/gal)									
Control Group	18	15	19	21	20	17	20	21	17	20
Treatment Group	19	24	21	22	18	23	20	25	21	23

4. Find the mean mileage of the control group.
5. Find the mean mileage of the treatment group.
6. Find the experimental difference of the means.

Answers

K. 18.8

R. 80.4

T. 2.8

C. 8

O. 88.4

E. 21.6

1	2	3	4	5	6
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**Chapter
11****Cumulative Review**

In Exercises 1–8, find the mean, median, and mode of the data set. Round your answer to the nearest tenth, when necessary.

1. 14, 1, 14, 10, 15, 18
2. 46, 47, 37, 49, 45, 46
3. 64, 73, 80, 63, 68, 80, 60
4. 40, 43, 45, 47, 47, 44, 43
5. 3, 17, 20, 8, 10, 19, 16
6. 15, 19, 20, 15, 14, 14, 11
7. 65, 57, 58, 61, 63, 60, 59, 54
8. 10, 1, 11, 7, 12, 11, 3, 10

In Exercises 9–14, find and interpret the standard deviation of the data set. Round your answer to the nearest tenth, when necessary.

9. 30, 29, 30, 20, 26, 22
10. 6, 7, 2, 9, 6, 3
11. 69, 53, 50, 51, 60, 54, 54
12. 18, 16, 17, 19, 14, 18, 17
13. 85, 82, 81, 87, 86, 85, 89, 85
14. 63, 62, 60, 60, 69, 70, 62, 65

In Exercises 15–20, tell whether x and y show *direct variation*, *inverse variation*, or *neither*.

15. $xy = 4$
16. $\frac{y}{7} = x$
17. $y = x + 4$
18. $x = y - 3$
19. $\frac{x}{3} = y$
20. $y = -4x$

In Exercises 21–28, the variables x and y vary inversely. Use the given value to write an equation relating x and y . Then find y when $x = 3$.

21. $x = -1, y = -7$
22. $x = -10, y = 4$
23. $x = -7, y = 1$
24. $x = 5, y = -2$
25. $x = \frac{8}{9}, y = 7$
26. $x = 2, y = \frac{3}{4}$
27. $x = 4, y = \frac{1}{5}$
28. $x = \frac{2}{3}, y = 4$

29. The results for your test scores are as follows: 73, 95, 76, 99, 85, 92, 78, 72, 71, 87, 98, 81, 88, 92, 93, 79, 75, 95, 77, 71, 90, 77, 100, 75, 94, 89, 84. What is the mean, median, and mode of the test scores? Round your answers to the nearest tenth, where necessary.

Chapter 11

Cumulative Review (continued)

In Exercises 30–35, simplify the expression.

30. $\frac{3x}{6x^2 - 27x^5}$

31. $\frac{6x + 2x^4}{3x}$

32. $\frac{x^2 - 3x - 4}{x^2 + 6x + 5}$

33. $\frac{x^2 + 8x - 9}{x^2 + 3x - 4}$

34. $\frac{x^2 - 16x + 63}{x^2 - x - 42}$

35. $\frac{x^2 - 13x + 40}{x^2 - 5x - 24}$

In Exercises 36–41, find the product.

36. $\frac{2xy^4}{x^3y} \cdot \frac{y}{6x^2}$

37. $\frac{5x^3y}{xy^2} \cdot \frac{y^4}{30x^2y}$

38. $\frac{x(x-4)}{x+8} \cdot \frac{(x+8)(x-2)}{x^2}$

39. $\frac{x^2 - x - 6}{x^2 - 16x + 60} \cdot \frac{x^2 - 7x + 6}{x^2 - 4x + 3}$

40. $\frac{x^2 - 10x + 16}{x^2 + x - 72} \cdot \frac{x^2 + 17x + 72}{x^2 + 15x + 56}$

41. $\frac{x^2 + 16x + 55}{x^2 - 3x - 40} \cdot \frac{x^2 - 5x - 24}{x^2 + x - 110}$

In Exercises 42–51, find the sum or difference.

42. $\frac{12}{7x} + \frac{8}{7x}$

43. $\frac{x}{2x^2} - \frac{3}{2x^2}$

44. $\frac{8}{x-2} + \frac{3x}{x-2}$

45. $\frac{3x^2}{x+5} - \frac{1}{x+5}$

46. $\frac{x}{3x+4} - \frac{8x}{3x+4}$

47. $\frac{x^3}{x^2+1} + \frac{x^2}{x^2+1}$

48. $\frac{x+8}{x^2+3x-40} - \frac{x+2}{x+8}$

49. $\frac{x+6}{x-7} + \frac{x-8}{x^2-12x+35}$

50. $\frac{3x+7}{x-1} - \frac{x-9}{x^2-12x+11}$

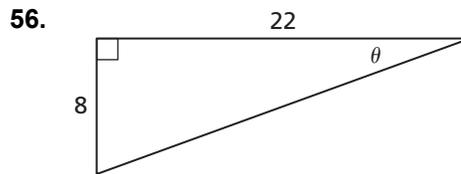
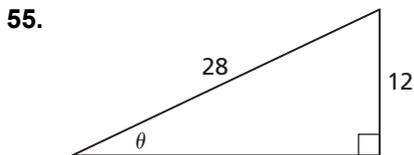
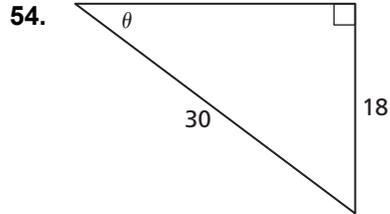
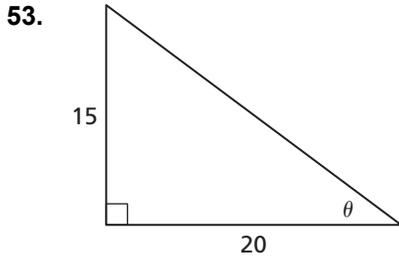
51. $\frac{x+11}{x^2-11x-12} + \frac{x+3}{x-12} - \frac{4}{x+1}$

52. The sides of a rectangle are $(2x + 7)$ inches and $(3x + 9)$ inches.

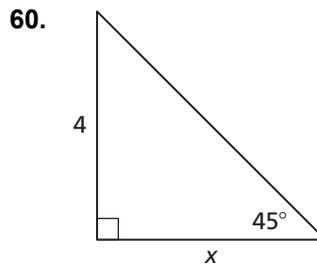
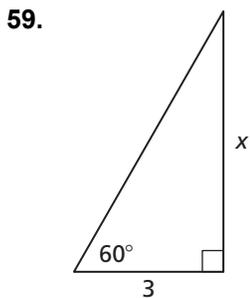
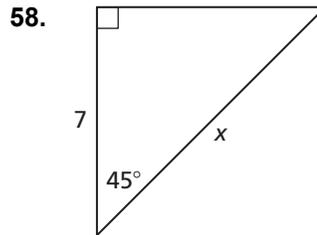
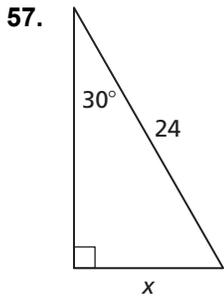
- a. Write an equation for the perimeter P of the rectangle.
- b. Write an equation for the area A of the rectangle.
- c. The value of x is 2. What is the perimeter and area of the rectangle?

Chapter 11 Cumulative Review (continued)

In Exercises 53–56, evaluate the six trigonometric functions of the angle θ .



In Exercises 57–60, find the value of x for the right triangle.



**Chapter
11****Cumulative Review (continued)**

In Exercises 61–66, find one positive angle and one negative angle that are coterminal with the given angle.

61. 88°

62. -305°

63. $-\frac{3\pi}{4}$

64. $\frac{5\pi}{6}$

65. 420°

66. $-\frac{4\pi}{3}$

In Exercises 67–72, convert the degree measure to radians or the radian measure to degrees.

67. 110°

68. 310°

69. $-\frac{4\pi}{3}$

70. $\frac{5\pi}{6}$

71. -60°

72. $-\frac{3\pi}{4}$

In Exercises 73–78, use the unit circle to evaluate the six trigonometric functions of θ .

73. 180°

74. 495°

75. $-\frac{\pi}{3}$

76. $\frac{\pi}{6}$

77. -120°

78. $\frac{7\pi}{4}$

In Exercises 79–90, graph the function.

79. $g(x) = 5 \sin x$

80. $g(x) = 2 \cos x$

81. $g(x) = \cos 3x$

82. $g(x) = \sin 4x$

83. $g(x) = \frac{1}{2} \sin 2x$

84. $g(x) = \frac{1}{4} \cos 2x$

85. $g(x) = \frac{1}{2} \sin 4\pi x$

86. $g(x) = 3 \cos 2\pi x$

87. $g(x) = \sin x - 1$

88. $g(x) = \cos x + 3$

89. $g(x) = 3 \sin 2x + 1$

90. $g(x) = \frac{1}{2} \cos x - 5$