11.1 -11.5 (skip 11.4)

Chapter 11 Data Analysis & Statistics

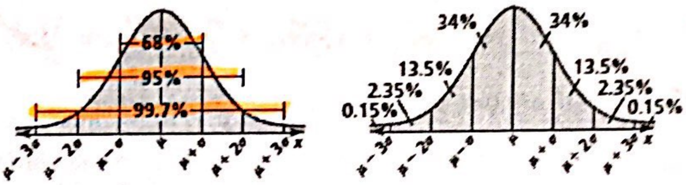
11.1 Using Normal Distributions

Core Concept

Areas Under a Normal Curve

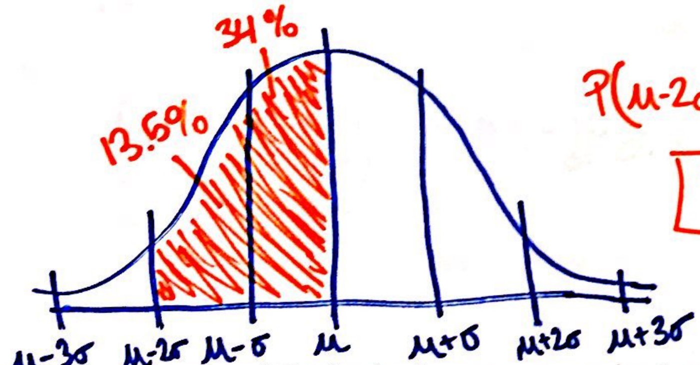
A normal distribution with mean μ (the Greek letter *mu*) and standard deviation σ (the Greek letter *sigma*) has these properties.

- The total area under the related normal curve is 1.
- About 68% of the area lies within 1 standard deviation of the mean.
- About 95% of the area lies within 2 standard deviations of the mean.
- About 99.7% of the area lies within 3 standard deviations of the mean.



****Concept 1: Finding Normal Probability****

1. A normal distribution has a mean μ and a standard deviation of σ. An x-value is randomly selected from the distribution. Find $P(\mu - 2\sigma \le x \le \mu)$.



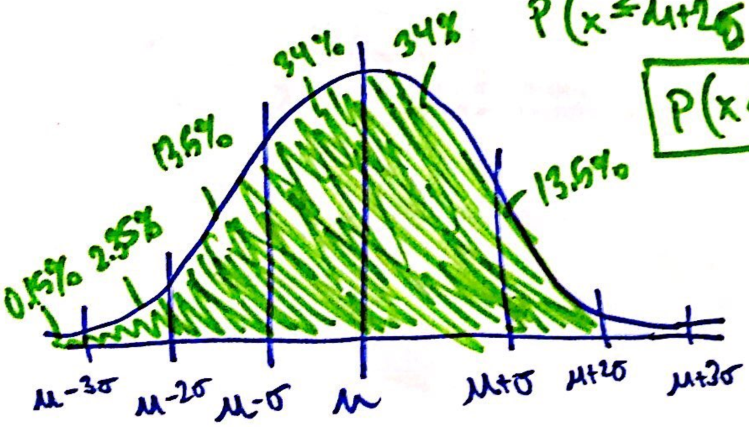
$P(\mu-2\sigma \le x \le \mu) = 13.5\% + 34\% = 47.5\%$

$\boxed{P(\mu-2\sigma \le x \le \mu) = 0.475}$

OR

$P = 0.135 + 0.34 = 0.475$

2. A normal distribution has a mean μ and a standard deviation of σ. An x-value is randomly selected from the distribution. Find $P(x \le \mu + 2\sigma)$.



$P(x \le \mu+2\sigma) = 0.15\% + 2.35\% + 13.5\% + 34\% + 34\% + 13.5\%$

$\boxed{P(x \le \mu+2\sigma) = 0.975}$

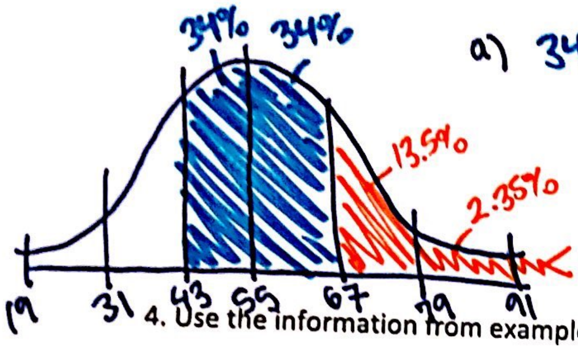
OR

$1 - (2.35\% + 0.15\%) = 1 - 0.025 = 0.975$

Concept 2: Interpreting Normally Distributed Data

3. The scores for a state's peace officer standards and training are normally distributed with a mean of 55 and a standard deviation of 12. The test scores range 0-100.

   a. About what percent of the people taking the test have scores between 43 and 67?
   b. An agency in the state will only hire applicants with test score of 67 or greater. About what percent of the people have test score that make them eligible to be hired by the agency?



a) 34% + 34% = $\boxed{68\%}$

b) 13.5% + 2.35% + 0.15% = $\boxed{16\%}$

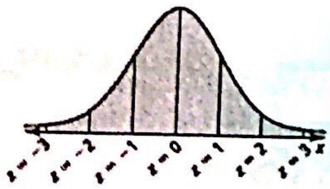
4. Use the information from example 2.

   a. About what percent of the people taking the test have scores between 31 and 55?
   b. An agency in the state will only hire applicants with test scores 79 percent or greater. About what percent of test takers are eligible to be hired by the agency?

a) 34% + 13.5% = 47.5%

b) 2.35% + 0.15% = 2.5%

The Standard Normal Distribution



The **standard normal distribution** is the normal distribution with mean 0 and standard deviation 1. The formula below can be used to transform x-values from a normal distribution with mean μ and standard deviation σ into z-values having a standard normal distribution.

Formula $z = \frac{x - \mu}{\sigma}$ — Subtract the mean from the given x-value, then divide by the standard deviation.

The z-value for a particular x-value is called the **z-score** for the x-value and is the number of standard deviations the x-value lies above or below the mean μ.

11.1 -11.5 (skip 11.4)

$\star$ $\boxed{\text{Z IS ALWAYS LESS THAN}}$

IF MORE THAN

$\star$ MUST TAKE $\boxed{1-P=}$

For a randomly selected z-value from a standard normal distribution, you can use the table below to find the probability that z is less than or equal to a given value. For example, the table shows that $P(z \le -0.4) = 0.3446$. You can find the value of $P(z \le -0.4)$ in the table by finding the value where row -0 and column $.4$ intersect.

$P(z \le 1.2)$

$P = 0.8849$

$P(z \le 2.3)$

$P = 0.9893$

$P(z \le 1.8)$

$P = 0.9641$

Standard Normal Table

z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
−3	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000+
−2	.0228	.0179	.0139	.0107	.0082	.0062	.0047	.0035	.0026	.0019
−1	.1587	.1357	.1151	.0968	.0808	.0668	.0548	.0446	.0359	.0287
−0	.5000	.4602	.4207	.3821	.3446	.3085	.2743	.2420	.2119	.1841
0	.5000	.5398	.5793	.6179	.6554	.6915	.7257	.7580	.7881	.8159
1	.8413	.8643	.8849	.9032	.9192	.9332	.9452	.9554	.9641	.9713
2	.9772	.9821	.9861	.9893	.9918	.9938	.9953	.9965	.9974	.9981
3	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000−

Concept 3: Using a z-score and the Standard Deviation Table

5. A study finds that the weights of infants at birth are normally distributed with a mean of 3270 grams and a standard deviation of 600 grams. An infant is randomly chosen. What is the probability that the infant weighs less than 4170 grams or less?

$$z = \frac{x-\mu}{\sigma} = \frac{4170-3270}{600} = 1.5$$

$$\boxed{P(z \le 1.5) = 0.9332}$$

6. A study finds that the weights of infants at birth are normally distributed with a mean of 3270 grams and a standard deviation of 600 grams. An infant is randomly chosen. What is the probability that the infant weighs at least 2790 grams?

MORE THAN

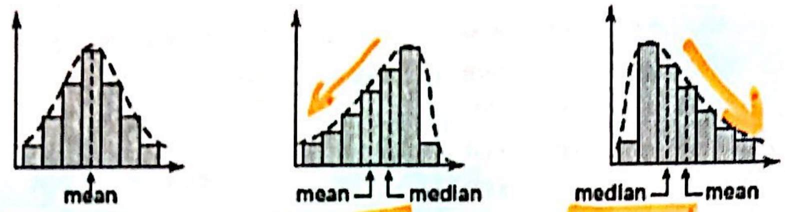
$$z = \frac{x-\mu}{\sigma} = \frac{2790-3270}{600} = -0.8$$

$$P(z \le -0.8) = 0.2119$$

$$\boxed{P = 1 - 0.2119 = 0.7881}$$

Recognizing Normal Distributions

Not all distributions are normal. For instance, consider the histograms shown below. The first histogram has a normal distribution. Notice that it is bell-shaped and symmetric. Recall that a distribution is symmetric when you can draw a vertical line that divides the histogram into two parts that are mirror images. Some distributions are skewed. The second histogram is *skewed left* and the third histogram is *skewed right*. The second and third histograms do *not* have normal distributions.



Bell-shaped and symmetric
- histogram has a normal distribution
- mean = median

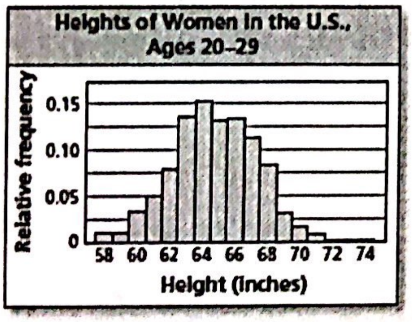
Skewed left
- histogram does not have a normal distribution
- mean < median

Skewed right
- histogram does not have a normal distribution
- mean > median

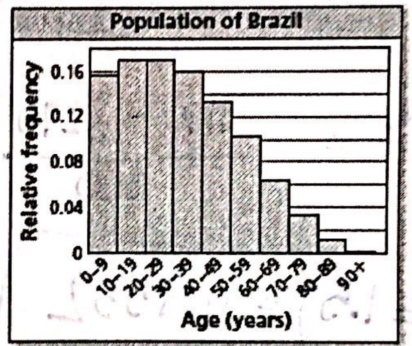
****Concept 4: Recognizing Normal Distribution****

7. Determine whether each histogram has a normal distribution.

a.



b.

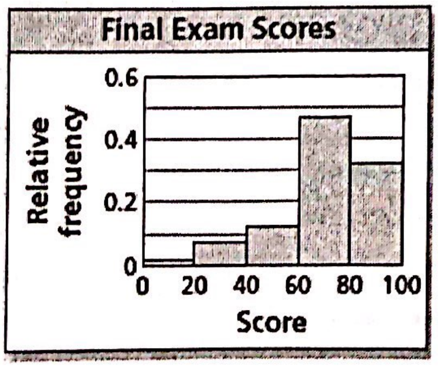


BELL SHAPED: NORMAL DIST.

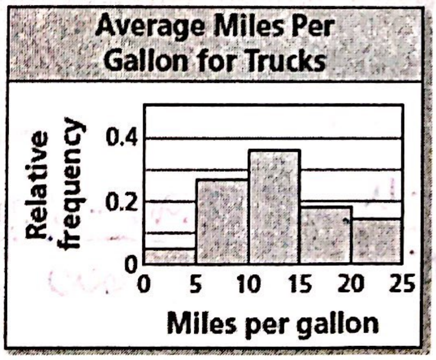
SKEWED RIGHT: NOT NORMAL DIST.

8. Determine whether each histogram has a normal distribution.

a.



b.



SKEWED LEFT:
NOT NORMAL DIST.

BELL CURVED:
NORMAL DIST.

11.5: Statistical Significance

Finding Margins of Error for Surveys

When conducting a survey, you need to make the size of your sample large enough so that it accurately represents the population. As the sample size increases, the *margin of error* decreases.

The **margin of error** gives a limit on how much the responses of the sample would differ from the responses of the population. For example, if 40% of the people in a poll favor a new tax law, and the margin of error is ±4%, then it is likely that between 36% and 44% of the entire population favor a new tax law.

Core Concept

Margin of Error Formula

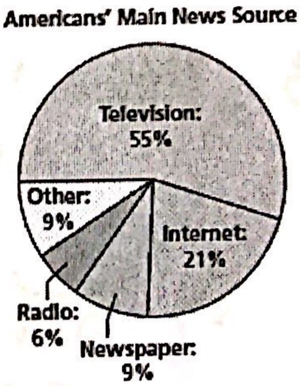
When a random sample of size n is taken from a large population, the margin of error is approximated by

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}}.$$

This means that if the percent of the sample responding a certain way is p (expressed as a decimal), then the percent of the population who would respond the same way is likely to be between $p - \frac{1}{\sqrt{n}}$ and $p + \frac{1}{\sqrt{n}}$.

CONCEPT 1: FINDING A MARGIN OF ERROR

1. In a survey of 2048 people in the U.S., 55% said that television is their main source of news. (a) What is the margin of error for the survey? (b) Give an interval that is likely to contain the exact percent of all people who use television as main source of news.



a) $\text{MoE} = \pm\frac{1}{\sqrt{2048}} = \pm 0.022 = 2.2\%$

b) $55\% - 2.2\% = 52.8\%$

$55\% + 2.2\% = 57.2\%$

$\boxed{\text{BETWEEN } 52.8\% \text{ \& } 57.2\%}$

2. In a survey of 2680 people in the U.S., 34% reported that movies are their main source of entertainment. a) What is the margin of error for the survey? (b) Give an interval that is likely to contain the exact percent of all people in the U.S. that movies are their main source of entertainment.

a) $M.O.E = \pm \frac{1}{\sqrt{2680}} = \pm 0.019 = 1.9\%$

b) $34\% - 1.9\% = 32.1\%$

$34\% + 1.9\% = 35.9\%$

$\boxed{\text{BETWEEN } 32.1\% \text{ \& } 35.9\%}$

3. a) $M.O.E = \pm \frac{1}{\sqrt{1028}} = \pm 0.031 = 3.1\%$

b) $87\% - 3.1\% = 83.9\%$

$87\% + 3.1\% = 90.1\%$

$\boxed{\text{BETWEEN } 83.9\% \text{ \& } 90.1\%}$