Chapter 4 Polynomial Functions

4.1: Graphing Polynomial Functions (pg. 158-161)

Recall that a monomial is a number, a variable, or the product of a number and one or more variables with whole number exponents. A polynomial is a monomial or a sum of monomials. A polynomial function is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. For this function, a_n is the leading coefficient, n is the degree, and a_0 is the constant term. A polynomial function is in *standard form* when its terms are written in descending order of exponents from left to right.

Common Polynomial Functions			
Degree	**Type**	**Standard Form**	**Example**
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^4 + 2x - 1$

CONCEPT 1: IDENTIFYING POLYNOMIAL FUNCTIONS

1. Decide whether each function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

a) $f(x) = 2x^3 + 5x + 8$ ← SF

D: 3 T: CUBIC LC: 2

b) $g(x) = -0.8x^3 + \sqrt{2}x^4 - 12$

SF $g(x) = \sqrt{2}x^4 - 0.8x^3 - 12$

D: 4 T: QUARTIC LC: $\sqrt{2}$

c) $h(x) = -x^2 + \cancel{7x^{-1}} + 4x$

$7x^{-1} = \frac{7}{x}$ NOT A POLYNOMIAL

d) $k(x) = x^2 + 3^x$

NOT A POLYNOMIAL

Decide whether each function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

2. $f(x) = 7 - 1.6x^2 - 5x$

SF $f(x) = -1.6x^2 - 5x + 7$

D: 2 T: QUADRATIC

LC: -1.6

3. $p(x) = x + 2x^{-2} + 9.5$

↑

NOT A POLY.

4. $q(x) = x^3 - 6x + 3x^4$

SF $q(x) = 3x^4 + x^3 - 6x$

D: 4 T: QUARTIC

LC: 3

****CONCEPT 2: EVALUATING A POLYNOMIAL FUNCTION****

5. Evaluate $f(x) = 2x^4 - 8x^2 + 5x - 7$ when $x = 3$.

$f(3) = 2(3)^4 - 8(3)^2 + 5(3) - 7$

$162 - 72 + 15 - 7$

$90 + 8$

98

$\boxed{f(3) = 98}$

6. Evaluate $f(x) = -2x^4 + 6x^3 - 3x + 11$ when $x = 4$.

$f(4) = -2(4)^4 + 6(4)^3 - 3(4) + 11$

$-512 + 384 - 12 + 11$

-129

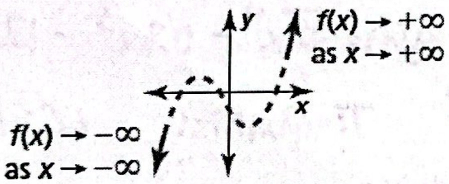
$\boxed{f(4) = -129}$

The **end behavior** of a function's graph is the behavior of the graph as x approaches positive infinity ($+\infty$) or negative infinity ($-\infty$). For the graph of a polynomial function, the end behavior is determined by the function's degree and the sign of its leading coefficient.

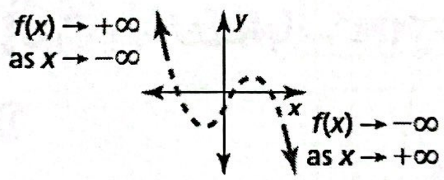
Core Concept

End Behavior of Polynomial Functions

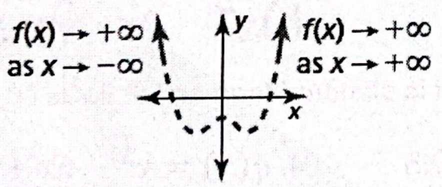
Degree: odd
Leading coefficient: positive



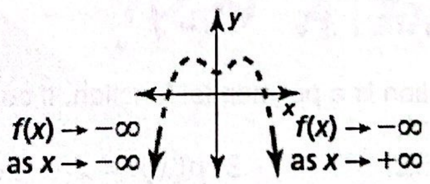
Degree: odd
Leading coefficient: negative



Degree: even
Leading coefficient: positive



Degree: even
Leading coefficient: negative



****CONCEPT 3: DESCRIBING END BEHAVIOR****

7. Describe the end behavior of the graph Evaluate $f(x) = -0.5x^4 + 2.5x^2 + x - 1$

D: 4 EVEN LC: −0.5 NEGATIVE (−)

$f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to +\infty$

8. Describe the end behavior of the graph Evaluate $f(x) = -0.3x^3 + 1.7x^2 - 4x + 6$

D: 3 ODD LC: -0.3 (-)

$f(x) \to +\infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to +\infty$

↑ ← ↓ →

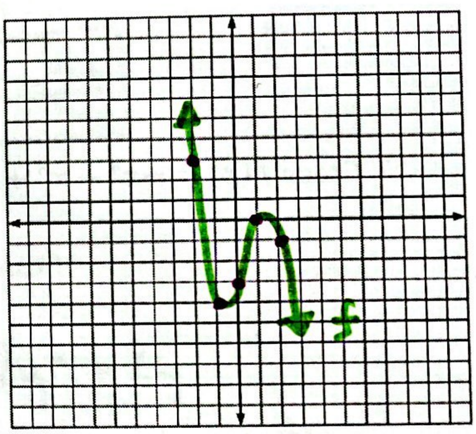
Graphing Polynomial Functions

To graph a polynomial function, first plot points to determine the shape of the graph's middle portion. Then connect the points with a smooth continuous curve and use what you know about end behavior to sketch the graph.

*****CONCEPT 4: GRAPHING POLYNOMIAL FUNCTIONS*****

10. Graph (a) $f(x) = -x^3 + x^2 + 3x - 3$ D: 3 ODD LC: -1 (-)

x	-2	-1	0	1	2
y	3	-4	-3	0	-1



$f(-2) = -(-2)^3 + (-2)^2 + 3(-2) - 3 = 8 + 4 - 6 - 3 = 3$ (-2, 3)

$f(-1) = -(-1)^3 + (-1)^2 + 3(-1) - 3 = 1 + 1 - 3 - 3 = -4$

$f(0) = -\cancel{(0)^3} + \cancel{(0)^2} + 3\cancel{(0)} - 3 = -3$

$f(1) = -(1)^3 + (1)^2 + 3(1) - 3 = -1 + 1 + 3 - 3 = 0$

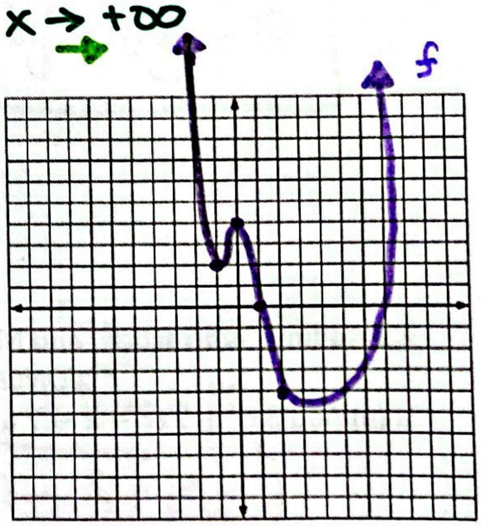
$f(2) = -(2)^3 + (2)^2 + 3(2) - 3 = -8 + 4 + 6 - 3 = -1$

$f(x) \to +\infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to +\infty$

↑ ← ↓ →

(b) $f(x) = x^4 - x^3 - 4x^2 + 4$

x	-2	-1	0	1	2
y	12	2	4	0	-4



$f(-2) = (-2)^4 - (-2)^3 - 4(-2)^2 + 4 = \cancel{16} + 8 - \cancel{16} + 4 = 12$

$f(-1) = (-1)^4 - (-1)^3 - 4(-1)^2 + 4 = 1 + 1 - \cancel{4} + \cancel{4} = 2$

$f(1) = (1)^4 - (1)^3 - 4(1)^2 + 4 = \cancel{1} - \cancel{1} - \cancel{4} + \cancel{4} = 0$

$f(2) = (2)^4 - (2)^3 - 4(2)^2 + 4 = \cancel{16} - 8 - \cancel{16} + 4 = -4$

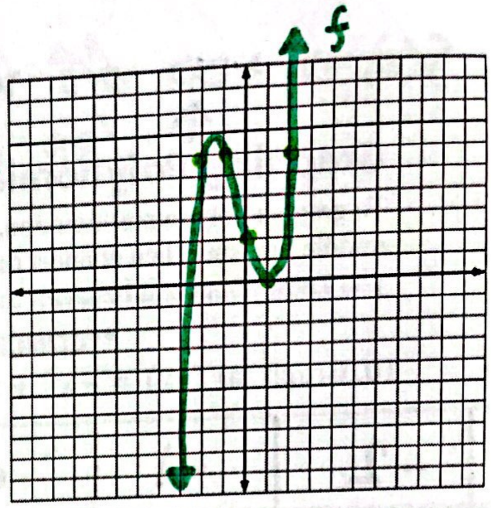
D: 4 EVEN LC: 1 (+)

$f(x) \to +\infty$ as $x \to -\infty$; $f(x) \to +\infty$ as $x \to +\infty$

↑ ← ↑ →

11. Graph $f(x) = x^3 + x^2 - 4x + 2$

x	-2	-1	0	1	2
f(x)	6	6	2	0	6



$f(-2) = (-2)^3 + (-2)^2 - 4(-2) + 2 = -8 + 4 + 8 + 2 = 6$

$f(-1) = (-1)^3 + (-1)^2 - 4(-1) + 2 = -1 + 1 + 4 + 2 = 6$

$f(1) = (1)^3 + (1)^2 - 4(1) + 2 = 1 + 1 - 4 + 2 = 0$

$f(2) = (2)^3 + (2)^2 - 4(2) + 2 = 8 + 4 - 8 + 2 = 6$

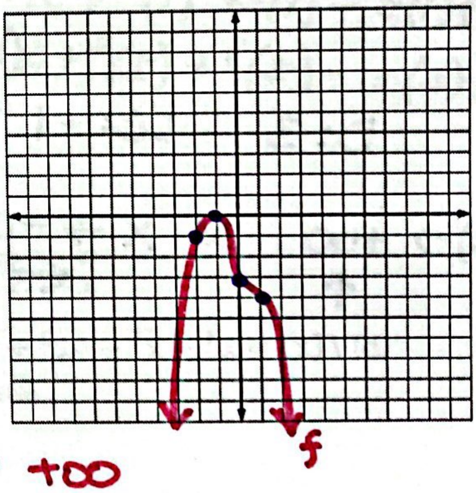
D: 3 LC: 1 ODD / POS.

$f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to +\infty$ as $x \to +\infty$

$\downarrow$ $\leftarrow$ $\uparrow$ $\rightarrow$

12. Graph $f(x) = -x^4 - x^3 + 2x^2 - x - 3$

X	-2	-1	0	1	2
f(x)	-1	0	-3	-4	-21



D: 4 LC: -1

EVEN / NEG.

$f(x) \to -\infty$ as $x \to -\infty$; $f(x) \to -\infty$ as $x \to +\infty$

$\downarrow$ $\leftarrow$ $\downarrow$ $\rightarrow$

4.2 Adding, Subtracting, and Multiplying Polynomials (pg. 166-169)

*****CONCEPT 1: ADDING POLYNOMIALS*****

1. Add $3x^3 + 2x^2 - x - 7$ and $x^3 - 10x^2 + 8$

$(3x^3 + 2x^2 - x - 7) + (1x^3 - 10x^2 + 8)$

$\boxed{4x^3 - 8x^2 - x + 1}$

****CONCEPT 2: SUBTRACTING POLYNOMIALS****

2.(a) $(8x^3 - 3x^2 - 2x + 9) - (2x^3 + 6x^2 - x + 1)$

$(8x^3 - 3x^2 - 2x + 9) + (-2x^3 - 6x^2 + x - 1)$

$\boxed{6x^3 - 9x^2 - x + 8}$

(b) Take $3z^2 + z - 4$ from $2z^2 + 3z$

$(2z^2 + 3z) + (-3z^2 - z + 4)$

$\boxed{-z^2 + 2z + 4}$

3. $(2x^2 - 6x + 5) + (7x^2 - x - 9)$

$\boxed{9x^2 - 7x - 4}$

4. $(3x^3 - 8x^2 - x - 4) + (-5x^3 + x^2 - 17)$

$\boxed{-2x^3 - 7x^2 - x - 21}$

****CONCEPT 3: MULTIPLYING POLYNOMIALS****

5. Multiply $y + 5$ and $3y^2 - 2y + 2$

$(y+5)(3y^2 - 2y + 2)$

$3y^3 - 2y^2 + 2y$

$15y^2 - 10y + 10$

$\boxed{3y^3 + 13y^2 - 8y + 10}$

6. Multiply $y - 2$ and $2y^2 - 3y + 5$

$(y-2)(2y^2 - 3y + 5)$

$2y^3 - 3y^2 + 5y$

$-4y^2 + 6y - 10$

$\boxed{2y^3 - 7y^2 + 11y - 10}$

****CONCEPT 4: MULTIPLYING THREE BINOMIALS****

7. Multiply $x - 1$, $x + 4$, and $x + 5$

$(x-1)(x+4)(x+5)$

$x^2+4x-x-4$

$(x^2+3x-4)(x+5)$

x^3+5x^2

$3x^2+15x$

$-4x-20$

$\boxed{x^3+8x^2+11x-20}$

8. Multiply $x + 2$, $x - 1$, and $x - 3$

$(x+2)(x-1)(x-3)$

$x^2-3x-x+3$

$(x+2)(x^2-4x+3)$

x^3-4x^2+3x

$2x^2-8x+6$

$\boxed{x^2-2x^2-5x+6}$

Core Concept

Special Product Patterns

Sum and Difference	Example
$(a+b)(a-b) = a^2 - b^2$	$(x+3)(x-3) = x^2 - 9$

Square of a Binomial	Example
$(a+b)^2 = a^2 + 2ab + b^2$	$(y+4)^2 = y^2 + 8y + 16$
$(a-b)^2 = a^2 - 2ab + b^2$	$(2t-5)^2 = 4t^2 - 20t + 25$

Cube of a Binomial	Example
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$	$(z+3)^3 = z^3 + 9z^2 + 27z + 27$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$	$(m-2)^3 = m^3 - 6m^2 + 12m - 8$

****CONCEPT 6: USING SPECIAL PRODUCT PATTERNS****

9. Find the product.

a) $(4n+5)(4n-5)$

$\boxed{16n^2-25}$

b) $(9y-2)^2$

$\boxed{81y^2-36y+4}$

c) $(ab+4)^3$

$(ab)^3+3(ab)^2(4)+3(ab)(4)^2+4^3$

$\boxed{a^3b^3+12a^2b^2+48ab+64}$

10. a) $(2n+7)(2n-7)$ b) $(4y+3)^2$ c) $(ab+2)^3$

$\boxed{4n^2-49}$

$\boxed{16y^2+24y+9}$

$(ab)^3+3(ab)^2(2)+3(ab)(2)^2+2^3$

$\boxed{a^3b^3+6a^2b^2+12ab+8}$

Pascal's Triangle

Consider the expansion of the binomial $(a+b)^n$ for whole number values of n. When you arrange the coefficients of the variables in the expansion of $(a+b)^n$, you will see a special pattern called **Pascal's Triangle.** Pascal's Triangle is named after French mathematician Blaise Pascal (1623–1662).

Core Concept

Pascal's Triangle

In Pascal's Triangle, the first and last numbers in each row are 1. Every number other than 1 is the sum of the closest two numbers in the row directly above it. The numbers in Pascal's Triangle are the same numbers that are the coefficients of binomial expansions, as shown in the first six rows.

	n	$(a+b)^n$	Binomial Expansion	Pascal's Triangle
0th row	0	$(a+b)^0 =$	1	1
1st row	1	$(a+b)^1 =$	$1a + 1b$	1 1
2nd row	2	$(a+b)^2 =$	$1a^2 + 2ab + 1b^2$	1 2 1
3rd row	3	$(a+b)^3 =$	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	1 3 3 1
4th row	4	$(a+b)^4 =$	$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$	1 4 6 4 1
5th row	5	$(a+b)^5 =$	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	1 5 10 10 5 1

CONCEPT 7: USING PASCAL'S TRIANGLE TO EXPAND BINOMIALS

11. Use Pascal's Triangle to expand (a) $(x-2)^5$ and (b) $(3y+1)^3$

a) $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$a=x$
$b=-2$

$(x-2)^5 = x^5 + 5(x)^4(-2) + 10(x)^3(-2)^2 + 10(x)^2(-2)^3 + 5x(-2)^4 + (-2)^5$

$\boxed{(x-2)^5 = x^5 - 10x^4 + 40x^3 - 80x^2 + 90x - 32}$

b) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$a=3y$
$b=1$

$(3y+1)^3 = (3y)^3 + 3(3y)^2(1) + 3(3y)(1)^2 + (1)^3$

$\boxed{(3y+1)^3 = 27y^3 + 27y^2 + 9y + 1}$

12. Use Pascal's Triangle to expand (a) $(x-3)^4$ and (b) $(2x+4)^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$a = x$
$b = -3$

$(x-3)^4 = x^4 + 4(x)^3(-3) + 6(x)^2(-3)^2 + 4(x)(-3)^3 + (-3)^4$

$\boxed{(x-3)^4 = x^4 - 12x^3 + 54x^2 - 108x + 81}$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$a = 2x$
$b = 4$

$(2x+4)^3 = (2x)^3 + 3(2x)^2(4) + 3(2x)(4)^2 + (4)^3$

$\boxed{(2x+4)^3 = 8x^3 + 48x^2 + 96x + 64}$

4.3: Dividing Polynomials (pg. 174-176)

Dividing Polynomials

- There are two methods for dividing polynomials. You will need to know both
  1. *Long Division*
  2. *Synthetic Division*

1. **LONG DIVISON:**

When you divide a polynomial $f(x)$ by a nonzero polynomial divisor $d(x)$, you get a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

NO x^2

****CONCEPT 1: USING POLYNOMIAL LONG DIVISION*****

1. Divide $2x^4 + 3x^3 + 5x - 1$ by $x^2 + 3x + 2$

$$\begin{array}{r l}
 & 2x^2 - 3x + 5 \\
x^2+3x+2 \overline{)} & 2x^4 + 3x^3 + 5x - 1 \\
 & -2x^4 \mp 6x^3 \mp 4x^2 \\
\hline
 & -3x^3 - 4x^2 + 5x - 1 \\
 & +3x^3 + 9x^2 + 6x \\
\hline
 & 5x^2 + 11x - 1 \\
 & -5x^2 \mp 15x + -10 \\
\hline
 & R\ -4x - 11
\end{array}$$

$2x^2 - 3x + 5 + \frac{-4x-11}{x^2+3x+2}$

$\boxed{2x^2 - 3x + 5 - \frac{4x+11}{x^2+3x+2}}$

2. $(x^3 - x^2 - 2x + 8) \div (x - 1)$

$$
\begin{array}{r|l}
 & x^2 \quad -2 \\
x-1 & \cancel{x^3} - \cancel{x^2} - 2x + 8 \\
 & -\cancel{x^3} + \cancel{x^2} \\
\hline
 & \quad -\cancel{2x} + 8 \\
 & \quad +\cancel{2x} + -2 \\
\hline
 & \quad R\ 6
\end{array}
$$

$$\boxed{x^2 - 2 + \frac{6}{x-1}}$$

3. $(x^3 + 2x^2 - x + 5) \div (x^2 - x + 1)$

$$
\begin{array}{r|l}
 & x \quad +3 \\
x^2 - x + 1 & \cancel{x^3} + 2x^2 - x + 5 \\
 & -\cancel{x^3} + x^2 + -x \\
\hline
 & \quad 3\cancel{x^2} - 2x + 5 \\
 & -\ 3\cancel{x^2} + 3x + -3 \\
\hline
 & \quad R\ x + 2
\end{array}
$$

$$\boxed{x + 3 + \frac{x+2}{x^2 - x + 1}}$$

2. SYNTHETIC DIVISION

- A shortcut for dividing polynomials by binomials of the form *(x - k)*

→<u>How to use</u> Synthetic Division:

*****CONCEPT 2: USING SYNTHETIC DIVISION*****

4. Divide $-x^3 + 4x^2 + 9$ by $x - 3$

0x

$$
\begin{array}{r|rrrr}
3 & -1 & 4 & 0 & 9 \\
 & \downarrow & -3 & 3 & 9 \\
\hline
 & -1_{x^2} & 1_{x^1} & 3_{x^0} & R\ 18
\end{array}
$$

$$\boxed{-x^2 + x + 3 + \frac{18}{x-3}}$$

5. Divide $3x^3 - 2x^2 + 2x - 5$ by $x + 1$

$$
\begin{array}{r|rrrr}
-1 & 3 & -2 & 2 & -5 \\
 & \downarrow & -3 & 5 & -7 \\
\hline
 & 3_{x^2} & -5_{x^1} & 7_{x^0} & R\ -12
\end{array}
$$

$$\boxed{3x^2 - 5x + 7 - \frac{12}{x+1}}$$

Divide using synthetic division

6. $(-x^3 + 3x^2 + x) \div (x - 2)$

$$\begin{array}{r|rrrr} 2 & -1 & 3 & 1 & 0 \\ & \downarrow & -2 & 2 & 6 \\ \hline & -1x^2 & 1x^1 & 3x^0 & R\,6 \end{array}$$

$$\boxed{-x^2 + x + 3 + \frac{6}{x-2}}$$

7. $(3x^3 - 7x^2 + 6x + 8) \div (x - 1)$

$$\begin{array}{r|rrrr} 1 & 3 & -7 & 6 & 8 \\ & \downarrow & 3 & -4 & 2 \\ \hline & 3x^2 & -4x^1 & 2x^0 & R\,10 \end{array}$$

$$\boxed{3x^2 - 4x + 2 + \frac{10}{x-1}}$$

Core Concept

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

- You can use the Remainder Theorem to tell whether *synthetic division* can be used to evaluate a polynomial function
- Your goal is to *determine the remainder of the function*

*****CONCEPT 3: EVALUATING A POLYNOMIAL*****

8. Use synthetic division to evaluate $f(x) = 5x^3 - x^2 + 13x + 29;\ x = -4$

$x = -4$
$+4 \quad +4$
$x + 4 = 0$

$$\begin{array}{r|rrrr} -4 & 5 & -1 & 13 & 29 \\ & \downarrow & -20 & 84 & -388 \\ \hline & 5 & -21 & 97 & R\,-359 \end{array}$$

$$\boxed{f(-4) = -359}$$

Use synthetic division to evaluate the function for the indicated value of x.

9. $f(x) = 4x^2 - 10x - 21; x = 5$

$$\begin{array}{r|rrr} 5 & 4 & -10 & -21 \\ & \downarrow & 20 & 50 \\ \hline & 4 & 10 & R\,29 \end{array}$$

$$\boxed{f(5) = 29}$$

10. $f(x) = 4x^3 - 2x^2 - 5x + 11; x = -2$

$$\begin{array}{r|rrrr} -2 & 4 & -2 & -5 & 11 \\ & \downarrow & -8 & 20 & -30 \\ \hline & 4 & -10 & 15 & R\,-19 \end{array}$$

$$\boxed{f(-2) = -19}$$

4.4: Factoring Polynomials (pg. 180-182)

Factoring Polynomials

- You've previously factored *Quadratic Equations* of degree 2
- You can also factor Polynomials with degree greater than 2

**Recall that a *factor* is a number that is multiplied to get a product

CONCEPT 1: FINDING A COMMON MONOMIAL FACTOR GCF

1. Factor each polynomial completely

a) $x^3 - 4x^2 - 5x$

$x(x^2-4x-5)$

$x(x-5)(x+1)$

b) $3x^5 - 48x^3$

$3x^3(x^2-16)$

$3x^3(x+4)(x-4)$

c) $5x^4 + 30x^3 + 45x^2$

$5x^2(x^2+6x+9)$

$5x^2(x+3)^2$

Factor each polynomial completely

2. $x^3 - 7x^2 + 10x$

$x(x^2-7x+10)$

$x(x-2)(x-5)$

3. $3x^7 - 75x^5$

$3x^5(x^2-25)$

$3x^5(x-5)(x+5)$

Core Concept

Special Factoring Patterns

Sum of Two Cubes	**Example**
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	$64x^3 + 1 = (4x)^3 + 1^3$
	$= (4x+1)(16x^2 - 4x + 1)$

Difference of Two Cubes	**Example**
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	$27x^3 - 8 = (3x)^3 - 2^3$
	$= (3x-2)(9x^2 + 6x + 4)$

CONCEPT 2: FACTORING THE SUM OR DIFFERENCE OF TWO CUBES

4. Factor (a) $x^3 - 125$ and (b) $16x^5 + 54x^2$ completely

a) $x^3 - 125$

$(x-5)(x^2+5x+25)$

b) $16x^5 + 54x^2$

$2x^2(8x^3+27)$

$2x^2(2x+3)(4x^2-6x+9)$

5. Factor (a) $x^3 - 64$ and (b) $-16x^5 - 250x^2$ completely

a) $x^3 - 64$

$(x-4)(x^2+4x+16)$

b) $-16x^5 - 250x^2$

$-2x^2(8x^3+125)$

$-2x^2(2x+5)(4x^2-10x+25)$

****CONCEPT 3: FACTORING BY GROUPING****

6. Factor $x^3 + 5x^2 - 4x - 20$ completely.

$x^2(x+5) - 4(x+5)$

$(x+5)(x^2-4)$

$(x+5)(x+2)(x-2)$

7. Factor $x^3 - 2x^2 - 9x + 18$ completely.

$x^2(x-2) - 9(x-2)$

$(x-2)(x^2-9)$

$(x-2)(x-3)(x+3)$

****CONCEPT 4: FACTORING POLYNOMIALS IN QUADRATIC FORM****

8. Factor (a) $16x^4 - 81$ and (b) $3x^8 + 15x^5 + 18x^2$ completely.

a) $16x^4 - 81$

$(4x^2+9)(4x^2-9)$

$(4x^2+9)(2x+3)(2x-3)$

b) $3x^8 + 15x^5 + 18x^2$

$3x^2(x^6+5x^3+6)$

$3x^2(x^3+2)(x^3+3)$

9. Factor (a) $625x^8 - 256$ and (b) $2x^{13} + 10x^9 + 8x^5$ completely.

a) $625x^8 - 256$

$(25x^4-16)(25x^4+16)$

$(5x^2-4)(5x^2+4)(25x^4+16)$

b) $2x^{13} + 10x^9 + 8x^5$

$2x^5(x^8+5x^4+4)$

$2x^5(x^4+4)(x^4+1)$

Core Concept

The Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

NEED R0

$x-2$

$k=2$

****CONCEPT 5: DETERMINING WHETHER A LINEAR BINOMIAL IS A FACTOR****

10. Determine whether (a) $x - 2$ is a factor of $f(x) = x^2 + 2x - 4$ and then (b) $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$

$$\begin{array}{r|rrr} 2 & 1 & 2 & -4 \\ & \downarrow & 2 & 8 \\ \hline & 1 & 4 & R4 \end{array}$$

NO!

b) $x+5$ $k=-5$

$$\begin{array}{r|rrrrr} -5 & 3 & 15 & -1 & 0 & 25 \\ & \downarrow & -15 & 0 & 5 & -25 \\ \hline & 3 & 0 & -1 & 5 & R0 \\ & x^3 & x^2 & x^1 & x^0 & \end{array}$$

YES!!!...

$3x^3 - x + 5$

****CONCEPT 6: FACTORING A BINOMIAL****

11. Show that $x + 3$ is a factor of $f(x) = x^4 + 3x^3 - x - 3$. Then factor completely.

$k=-3$

$$\begin{array}{r|rrrrr} -3 & 1 & 3 & 0 & -1 & -3 \\ & \downarrow & -3 & 0 & 0 & 3 \\ \hline & 1 & 0 & 0 & -1 & R0 \\ & x^3 & x^2 & x^1 & x^0 & \end{array}$$

$(x+3)(x^3-1)$

$(x+3)(x-1)(x^2+x+1)$

12. Show that $x - 2$ is a factor of $f(x) = x^4 - 2x^3 + x - 2$. Then factor completely.

$$\begin{array}{r|rrrrr} 2 & 1 & -2 & 0 & 1 & -2 \\ & \downarrow & 2 & 0 & 0 & 2 \\ \hline & 1 & \cancel{0} & \cancel{0} & 1 & R0 \\ & x^3 & \cancel{x^2} & \cancel{x^1} & x^0 & \end{array}$$

$(x-2)(x^3+1)$

$(x-2)(x+1)(x^2-x+1)$