

**9-5 Study Guide and Intervention****Base  $e$  and Natural Logarithms**

**Base  $e$  and Natural Logarithms** The irrational number  $e \approx 2.71828\dots$  often occurs as the base for exponential and logarithmic functions that describe real-world phenomena.

<b>Natural Base <math>e</math></b>	As $n$ increases, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.71828\dots$ $\ln x = \log_e x$
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The functions  $y = e^x$  and  $y = \ln x$  are inverse functions.

<b>Inverse Property of Base <math>e</math> and Natural Logarithms</b>	$e^{\ln x} = x$ $\ln e^x = x$
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Natural base expressions can be evaluated using the  $e^x$  and  $\ln$  keys on your calculator.

**Example 1 Evaluate  $\ln 1685$ .**

Use a calculator.

$$\ln 1685 \approx 7.4295$$

**Example 2 Write a logarithmic equation equivalent to  $e^{2x} = 7$ .**

$$e^{2x} = 7 \rightarrow \log_e 7 = 2x \text{ or } 2x = \ln 7$$

**Example 3 Evaluate  $\ln e^{18}$ .**

Use the Inverse Property of Base  $e$  and Natural Logarithms.

$$\ln e^{18} = 18$$

**Exercises**

Use a calculator to evaluate each expression to four decimal places.

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|-----------------|-----------------|------------------|------------------|
| 1. $\ln 732$    | 2. $\ln 84,350$ | 3. $\ln 0.735$   | 4. $\ln 100$     |
| 5. $\ln 0.0824$ | 6. $\ln 2.388$  | 7. $\ln 128,245$ | 8. $\ln 0.00614$ |

Write an equivalent exponential or logarithmic equation.

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|---------------------|---------------------|---------------------|----------------------|
| 9. $e^{15} = x$     | 10. $e^{3x} = 45$   | 11. $\ln 20 = x$    | 12. $\ln x = 8$      |
| 13. $e^{-5x} = 0.2$ | 14. $\ln(4x) = 9.6$ | 15. $e^{8.2} = 10x$ | 16. $\ln 0.0002 = x$ |

Evaluate each expression.

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|---------------|------------------|-------------------|--------------------|
| 17. $\ln e^3$ | 18. $e^{\ln 42}$ | 19. $e^{\ln 0.5}$ | 20. $\ln e^{16.2}$ |
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