

9-2 Study Guide and Intervention *(continued)*

Logarithms and Logarithmic Functions

Solve Logarithmic Equations and Inequalities

Logarithmic to Exponential Inequality	If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$. If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$.
Property of Equality for Logarithmic Functions	If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.
Property of Inequality for Logarithmic Functions	If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.

Example 1 Solve $\log_2 2x = 3$.

$$\begin{aligned} \log_2 2x &= 3 && \text{Original equation} \\ 2x &= 2^3 && \text{Definition of logarithm} \\ 2x &= 8 && \text{Simplify.} \\ x &= 4 && \text{Simplify.} \end{aligned}$$

The solution is $x = 4$.

Example 2 Solve $\log_5 (4x - 3) < 3$.

$$\begin{aligned} \log_5 (4x - 3) &< 3 && \text{Original equation} \\ 0 < 4x - 3 &< 5^3 && \text{Logarithmic to exponential inequality} \\ 3 < 4x &< 125 + 3 && \text{Addition Property of Inequalities} \\ \frac{3}{4} < x &< 32 && \text{Simplify.} \end{aligned}$$

The solution set is $\left\{x \mid \frac{3}{4} < x < 32\right\}$.

Exercises

Solve each equation or inequality.

1. $\log_2 32 = 3x$

2. $\log_3 2c = -2$

3. $\log_{2x} 16 = -2$

4. $\log_{25} \left(\frac{x}{2}\right) = \frac{1}{2}$

5. $\log_4 (5x + 1) = 2$

6. $\log_8 (x - 5) = \frac{2}{3}$

7. $\log_4 (3x - 1) = \log_4 (2x + 3)$

8. $\log_2 (x^2 - 6) = \log_2 (2x + 2)$

9. $\log_{x+4} 27 = 3$

10. $\log_2 (x + 3) = 4$

11. $\log_x 1000 = 3$

12. $\log_8 (4x + 4) = 2$

13. $\log_2 2x > 2$

14. $\log_5 x > 2$

15. $\log_2 (3x + 1) < 4$

16. $\log_4 (2x) > -\frac{1}{2}$

17. $\log_3 (x + 3) < 3$

18. $\log_{27} 6x > \frac{2}{3}$