9-1 Study Guide and Intervention (continued)

Exponential Functions

Exponential Equations and Inequalities All the properties of rational exponents that you know also apply to real exponents. Remember that $a^m \cdot a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, and $a^m \div a^n = a^{m-n}$.

Property of Equality for Exponential Functions	If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.
Property of Inequality for Exponential Functions	If $b > 1$ then $b^x > b^y$ if and only if $x > y$ and $b^x < b^y$ if and only if $x < y$.

Example 1 Solve $4^{x-1} = 2^{x+5}$.

$$4^{x-1} = 2^{x+5} \quad \text{Original equation}$$
$$(2^2)^{x-1} = 2^{x+5} \quad \text{Rewrite 4 as } 2^2.$$
$$2(x-1) = x+5 \quad \text{Prop. of Inequality for Exponential Functions}$$
$$2x - 2 = x + 5 \quad \text{Distributive Property}$$
$$x = 7 \quad \text{Subtract } x \text{ and add 2 to each side.}$$

Example 2 Solve $5^{2x-1} > \dfrac{1}{125}$.

$$5^{2x-1} > \frac{1}{125} \quad \text{Original inequality}$$
$$5^{2x-1} > 5^{-3} \quad \text{Rewrite } \tfrac{1}{125} \text{ as } 5^{-3}.$$
$$2x - 1 > -3 \quad \text{Prop. of Inequality for Exponential Functions}$$
$$2x > -2 \quad \text{Add 1 to each side.}$$
$$x > -1 \quad \text{Divide each side by 2.}$$

The solution set is $\{x \mid x > -1\}$.

Exercises

Simplify each expression.

1. $\left(3^{\sqrt{2}}\right)^{\sqrt{2}}$

2. $25^{\sqrt{2}} \cdot 125^{\sqrt{2}}$

3. $\left(x^{\sqrt{2}} y^{3\sqrt{2}}\right)^{\sqrt{2}}$

4. $\left(x^{\sqrt{6}}\right)\left(x^{\sqrt{5}}\right)$

5. $\left(x^{\sqrt{6}}\right)^{\sqrt{5}}$

6. $(2x^{\pi})(5x^{3\pi})$

Solve each equation or inequality. Check your solution.

7. $3^{2x-1} = 3^{x+2}$

8. $2^{3x} = 4^{x+2}$

9. $3^{2x-1} = \dfrac{1}{9}$

10. $4^{x+1} = 8^{2x+3}$

11. $8^{x-2} = \dfrac{1}{16}$

12. $25^{2x} = 125^{x+2}$

13. $4^{\sqrt{x}} = 16^{\sqrt{5}}$

14. $x^{\sqrt{3}} = 36^{\sqrt{\frac{3}{4}}}$

15. $x^{\sqrt{2}} = 81^{\frac{1}{\sqrt{8}}}$

16. $3^{x-4} < \dfrac{1}{27}$

17. $4^{2x-2} > 2^{x+1}$

18. $5^{2x} < 125^{x-5}$

19. $10^{4x+1} > 100^{x-2}$

20. $7^{3x} < 49^{x^2}$

21. $8^{2x-5} < 4^{x+8}$

Glencoe Algebra