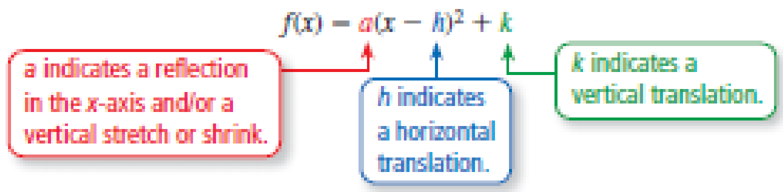
Chapter 2 Quadratic Functions

2.1: Transformations of Quadratic Functions (pg. 48 - 51)

Writing Transformations of Quadratic Functions

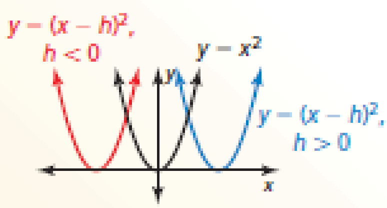
The lowest point on a parabola that opens up or the highest point on a parabola that opens down is the **vertex**. The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a \neq 0$ and the vertex is (h, k).



Core Concept

Horizontal Translations

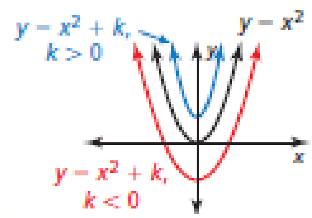
$$f(x) = x^2$$
$$f(x - h) = (x - h)^2$$



- shifts left when $h < 0$
- shifts right when $h > 0$

Vertical Translations

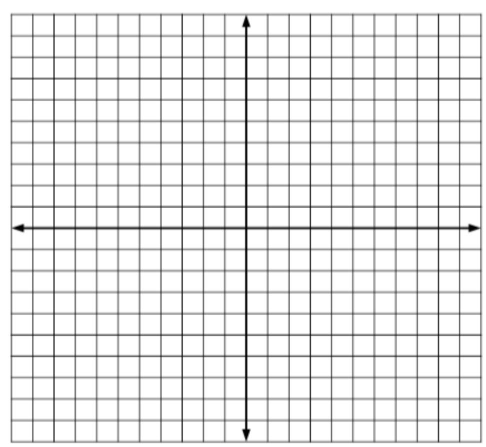
$$f(x) = x^2$$
$$f(x) + k = x^2 + k$$



- shifts down when $k < 0$
- shifts up when $k > 0$

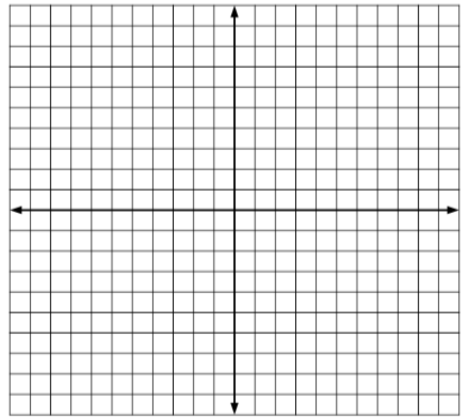
*****CONCEPT 1: TRANSLATIONS OF A QUADRATIC FUNCTION*****

1. Describe the transformation of $f(x) = x^2$ represented by $g(x) = (x + 4)^2 - 1$. Then graph each function. (tip: describe the transformation before your graph.)



2.1 - 2.4 (skip 2.3)

2. Describe the transformation of $f(x) = x^2$ represented by $h(x) = (x-1)^2 + 2$. Then graph each function.

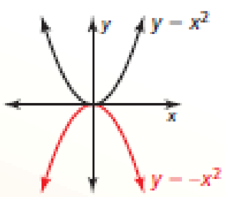


Core Concept

Reflections in the *x*-Axis

$f(x) = x^2$

$-f(x) = -(x^2) = -x^2$

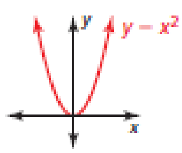


flips over the *x*-axis

Reflections in the *y*-Axis

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

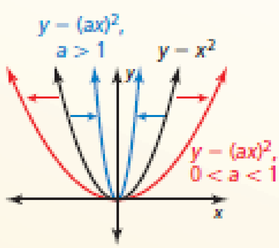


$y = x^2$ is its own reflection in the y-axis.

Horizontal Stretches and Shrinks

$f(x) = x^2$

$f(ax) = (ax)^2$

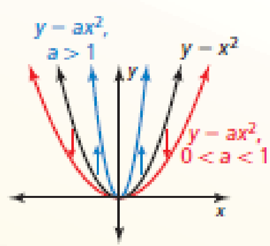


- horizontal stretch (away from y-axis) when $0 < a < 1$
- horizontal shrink (toward y-axis) when $a > 1$

Vertical Stretches and Shrinks

$f(x) = x^2$

$a \cdot f(x) = ax^2$



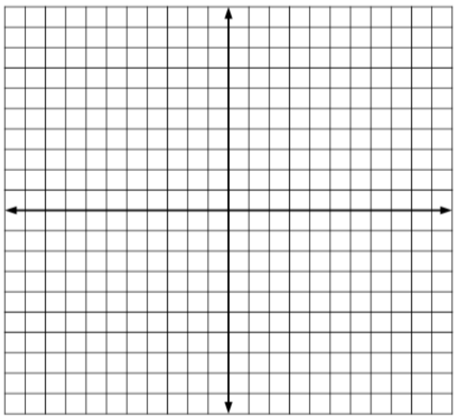
- vertical stretch (away from *x*-axis) when $a > 1$
- vertical shrink (toward *x*-axis) when $0 < a < 1$

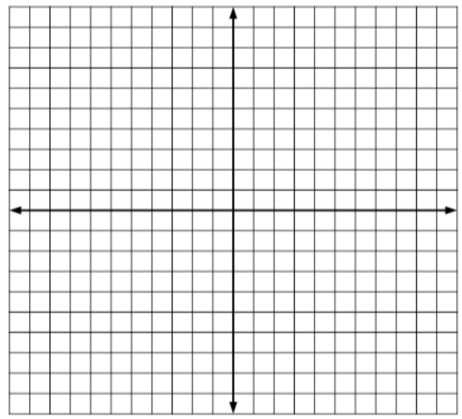
*****CONCEPT 2: TRANSFORMATIONS OF QUADRATIC FUNCTIONS*****

3. Describe the transformation of $f(x) = x^2$ represented by g. Then graph each function.

a. $g(x) = -\frac{1}{2}x^2$

b. $g(x) = (2x)^2 + 1$

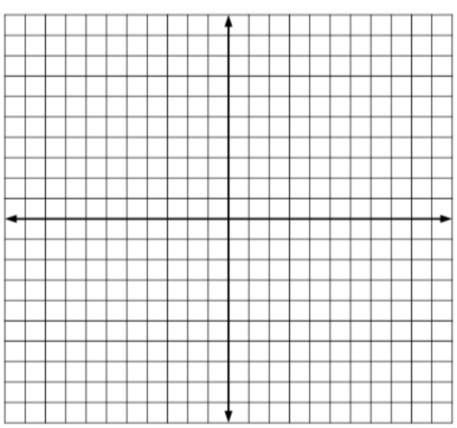


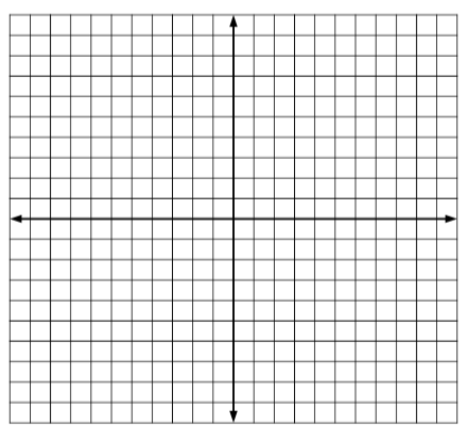


4. Describe the transformation of $f(x) = x^2$ represented by *h*. Then graph each function.

a. $h(x) = -3x^2$

b. $h(x) = \left(\frac{1}{4}x\right)^2 - 2$





*****CONCEPT 3: WRITING A TRANSFORMED QUADRATIC FUNCTION*****

5. Let the graph of g be a vertical stretch by a factor of 2 and a reflection in the x-axis, followed by a translation 3 units down of the graph of $f(x) = x^2$. Write a rule for g and identify the vertex.

6. Let the graph of g be a horizontal shrink by a factor of $\frac{1}{3}$ and a reflection in the y-axis, followed by a translation 2 units up of the graph of $f(x) = x^2$. Write a rule for g and identify the vertex.

*****CONCEPT 4: WRITING OTHER TRANSFORMED QUADRATIC FUNCTIONS*****

7. Let the graph of g be a translation 3 units right and 2 units up, followed by a reflection in the y-axis of the graph of $f(x) = x^2 - 5x$. Write a rule for *g.*

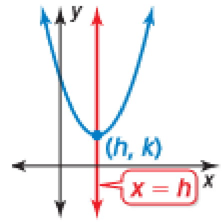
8. Let the graph of g be a translation 4 units left and 1 unit down, followed by a reflection in the y-axis of the graph of $f(x) = 2x^2 + x$. Write a rule for g.

2.1 - 2.4 (skip 2.3)

2.2: Characteristics of Quadratic Functions (pg. 56-60)

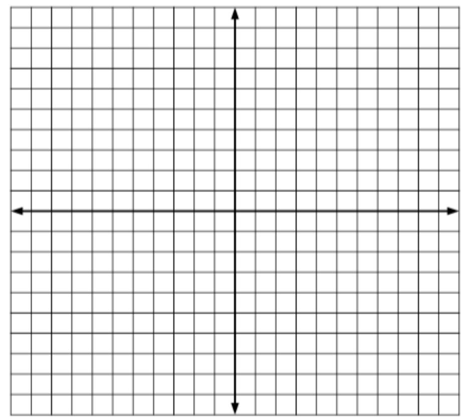
Exploring Properties of Parabolas

An **axis of symmetry** is a line that divides a parabola into mirror images and passes through the vertex. Because the vertex of $f(x) = a(x - h)^2 + k$ is (h, k), the axis of symmetry is the vertical line $x = h$.

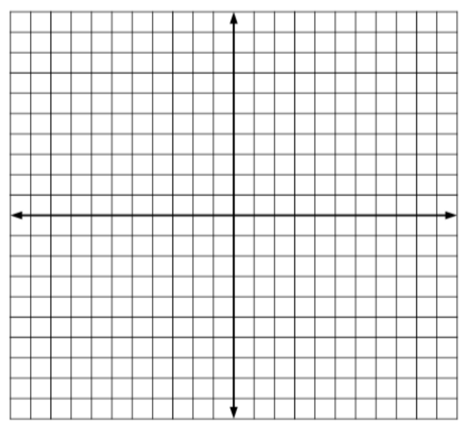


***** CONCEPT 1: GRAPHING VERTEX FORM WITH RELATION TO SYMMETRY*****

1. Graph $f(x) = -2(x+3)^2 + 4$. Label the vertex and axis of symmetry.



2. Graph $f(x) = 0.5(x+4)^2 - 2$. Label the vertex and axis of symmetry.

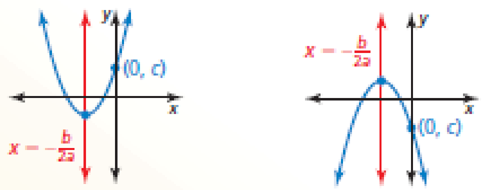


2.1 - 2.4 (skip 2.3)

Core Concept

Properties of the Graph of $f(x) = ax^2 + bx + c$

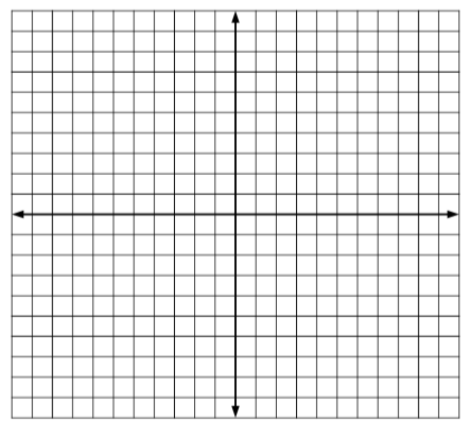
$y = ax^2 + bx + c, a > 0$ $\quad\quad$ $y = ax^2 + bx + c, a < 0$



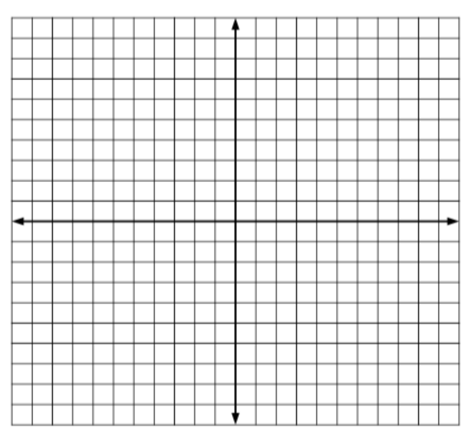
- The parabola opens up when $a > 0$ and opens down when $a < 0$.
- The graph is narrower than the graph of $f(x) = x^2$ when $|a| > 1$ and wider when $|a| < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- The y-intercept is c. So, the point $(0, c)$ is on the parabola.

***** CONCEPT 2: GRAPHING QUADRATICS IN STANDARD FORM*****

3. Graph $f(x) = 3x^2 - 6x + 1$. Label the vertex and the axis of symmetry.



4. Graph $f(x) = 0.5x^2 - 4x - 2$. Label the vertex and axis of symmetry.

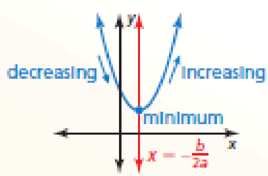


Core Concept

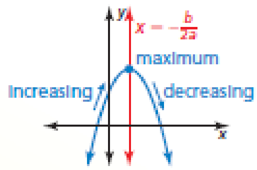
Minimum and Maximum Values

For the quadratic function $f(x) = ax^2 + bx + c$, the y-coordinate of the vertex is the **minimum value** of the function when $a > 0$ and the **maximum value** when $a < 0$.

$a > 0$



$a < 0$



$a > 0$

- Minimum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \geq f\left(-\frac{b}{2a}\right)$
- Decreasing to the left of $x = -\frac{b}{2a}$
- Increasing to the right of $x = -\frac{b}{2a}$

$a < 0$

- Maximum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \leq f\left(-\frac{b}{2a}\right)$
- Increasing to the left of $x = -\frac{b}{2a}$
- Decreasing to the right of $x = -\frac{b}{2a}$

***** CONCEPT 3: FINDING A MAXIMUM OR MINIMUM VALUE*****

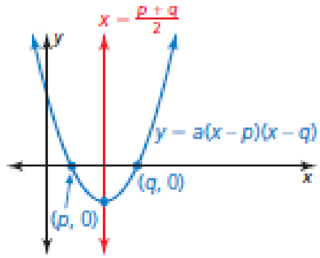
5. Find the minimum value or maximum value of $f(x) = \frac{1}{2}x^2 - 2x - 1$. Describe the domain and range of the function.

6. Find the minimum and maximum value of $f(x) = 2x^2 + 8x - 6$. Describe the domain and range of the function.

Core Concept

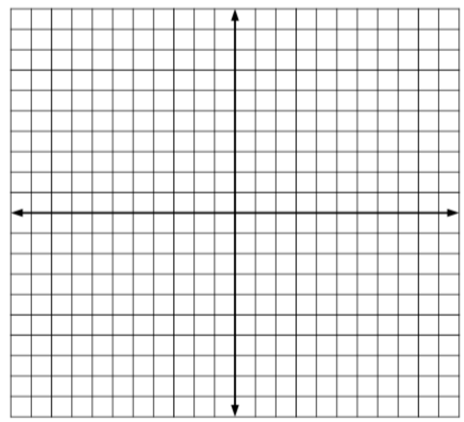
Properties of the Graph of $f(x) = a(x - p)(x - q)$

- Because $f(p) = 0$ and $f(q) = 0$, p and q are the x-intercepts of the graph of the function.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$. So, the axis of symmetry is $x = \frac{p+q}{2}$.
- The parabola opens up when $a > 0$ and opens down when $a < 0$.

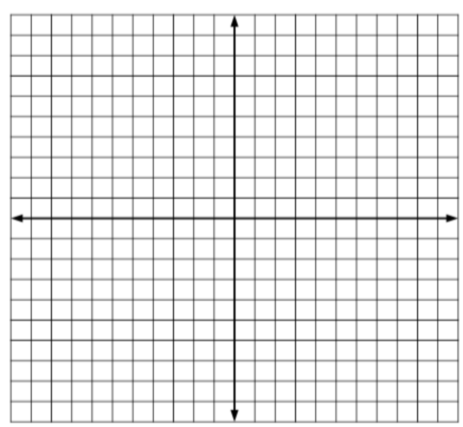


***** CONCEPT 4: GRAPHING A QUADRATIC IN INTERCEPT FORM*****

7. Graph $f(x) = -2(x+3)(x-1)$. Label the x-intercepts, vertex, and axis of symmetry.

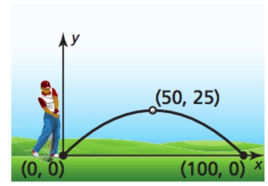


8. Graph $f(x) = -\frac{1}{3}(x-4)(x+2)$. Label the x-intercepts, vertex, and axis of symmetry.



***** CONCEPT 5: WORD PROBLEMS*****

9. The parabola shows the path of your first golf shot, where *x* is the horizontal distance (in yards) and *y* is the corresponding height (in yards). The path of your second shot can be modeled by the function $f(x) = -0.02x(x - 80)$. Which shot travels farther before hitting the ground? Which travels higher?



10. Use the graph from the last problem. The path of your third golf shot is $g(x) = -0.03x(x - 40)$. Does your first or third shot travel farther before hitting the ground? Which travels higher.

2.4: Modeling with Quadratic Functions (pg. 76-79)

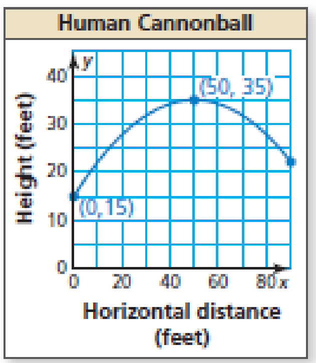
Core Concept

Writing Quadratic Equations

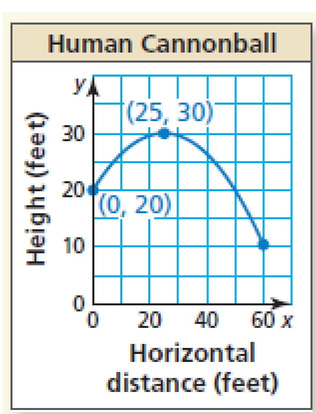
Given a point and the vertex (h, k)	Use vertex form: $y = a(x - h)^2 + k$
Given a point and x-intercepts p and q	Use intercept form: $y = a(x - p)(x - q)$
Given three points	Write and solve a system of three equations in three variables.

***** CONCEPT 1: GIVEN AN EQUATION & A POINT*****

1. The graph shows the parabolic path of a performer who is shot out of cannon, where *y* is the height (in feet) and *x* is the horizontal distance traveled (in feet). Write an equation of the parabola. The performer lands in a net 90 feet from the cannon. What is the height of the net to the nearest foot?

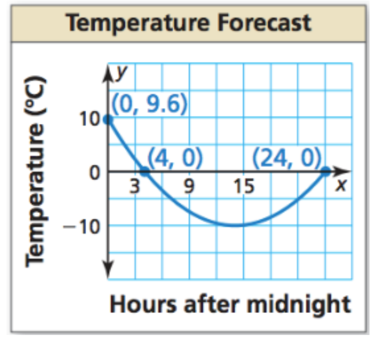


2. The graph shows the parabolic path of a performer who is shot out of cannon, where *y* is the height (in feet) and *x* is the horizontal distance traveled (in feet). Write an equation of the parabola. The performer lands in a net 60 feet from the cannon. What is the height of the net to the nearest foot?



***** CONCEPT 2: GIVEN A POINT & X-INTERCEPTS*****

3. A meteorologist creates a parabola to predict the temperature tomorrow, where *x* is the number of hours after midnight and *y* is the temperature (in degrees Celsius). Write a function *f* that models the temperature over time. What is the coldest temperature?



4. A meteorologist creates a parabola to predict the temperature tomorrow, where *x* is the number of hours after midnight and *y* is the temperature (in degrees Celsius). Write a function *f* that models the temperature over time. What is the coldest temperature?