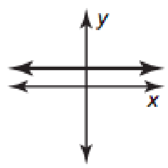
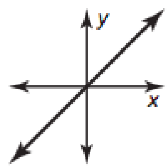
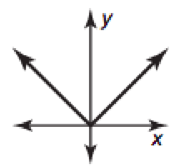
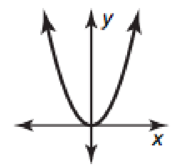
Chapter 1 Linear Functions

1.1 Parent Functions and Transformations (pg. 4-7)

The **parent function** is the most basic function in a family. Functions in the same family are *transformations* of their parent function.

Parent Functions

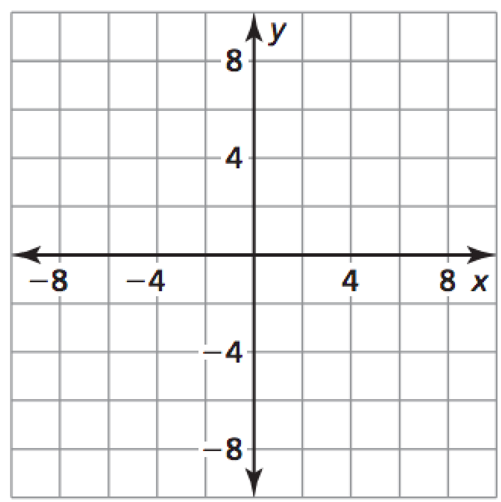
Family	Constant	Linear	Absolute Value	Quadratic
Rule	$f(x) = 1$	$f(x) = x$	$f(x) = \lvert x \rvert$	$f(x) = x^2$
Graph				
Domain	All real numbers	All real numbers	All real numbers	All real numbers
Range	$y = 1$	All real numbers	$y \geq 0$	$y \geq 0$

A **transformation** changes the size, shape, position, or orientation of a graph.
A **translation** is a transformation that shifts a graph horizontally and/or vertically but does not change its size, shape, or orientation.

*****CONCEPT 1: GRAPHING & DESCRIBING TRANSLATIONS*****

1. Graph $g(x) = |x + 6|$ and its parent function. Then describe the transformation.

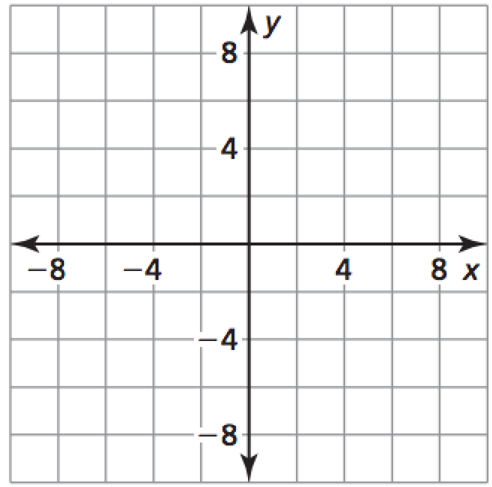
x	*f(x) = |x|*	*g(x) = |x + 2|*
-4		
-2		
0		
2		
4		



Describe the transformation: ____

2. Graph $g(x) = x - 4$ and its parent function. Then describe the transformation.

x	***f(x) = x***	***g(x) = x - 4***
-4		
-2		
0		
2		
4		

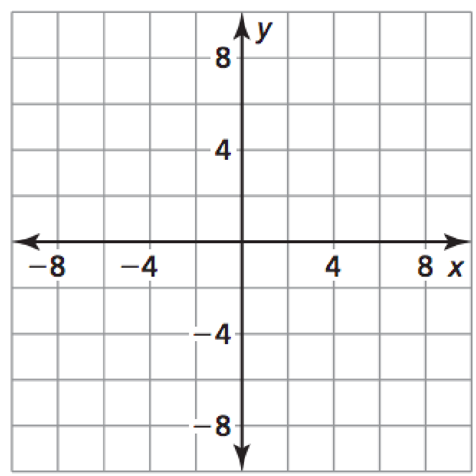


Describe the transformation____

A **reflection** is a transformation that flips a graph over a line called the *line of reflection*. A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

*****CONCEPT 2: GRAPHING & DESCRIBING REFLECTIONS*****

3. Graph $p(x) = -x^2$ and its parent function. Then describe the transformation.

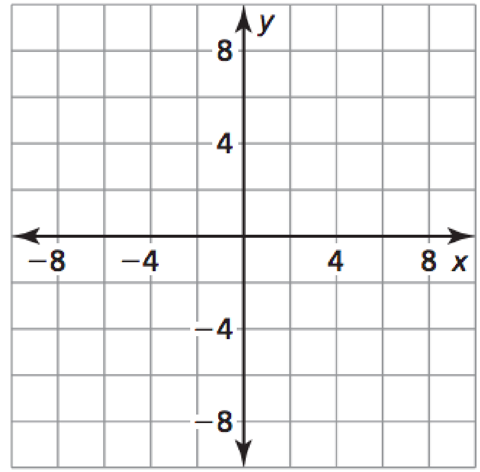


x	***f(x) = x²***	***p(x) = -x²***
-2		
-1		
0		
1		
2		

Describe the transformation:____

4. Graph $k(x) = -x$ and its parent function. Then describe the transformation.

x	***f(x) = x***	***k(x) = -x***
-2		
-1		
0		
1		
2		



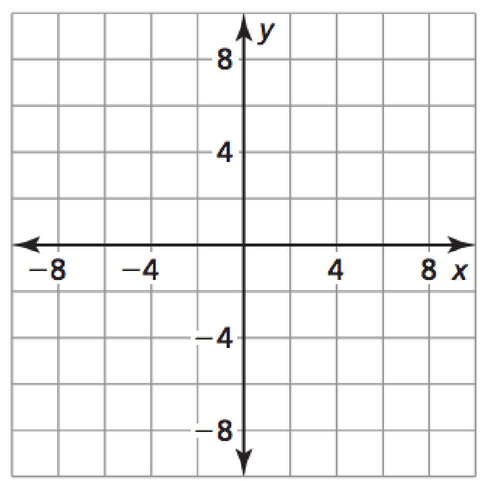
Describe the transformation: ____

When the factor is greater than 1($x > 1$), the transformation is a **vertical stretch**. When the factor is greater than 0 and less than 1 or ($0 < x < 1$), it is a **vertical shrink**.

*****CONCEPT 3: GRAPHING & DESCRIBING STRETCHES AND SHRINKS*****

5. Graph $g(x) = 2|x|$ and its parent function. Then describe the transformation.

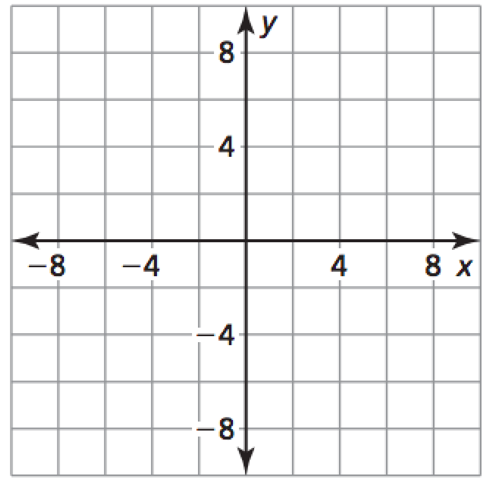
x	***f(x) = |x|***	***g(x) = 2|x|***
-2		
-1		
0		
1		
2		



Describe the transformation*:* ____

6. Graph $p(x) = \frac{1}{2}x^2$ and its parent function. Then describe the transformation.

x	***f(x) = x²***	***p(x) = $\frac{1}{2}x^2$***
-2		
-1		
0		
1		
2		



Describe the transformation: ____

*****CONCEPT 4: MODELING WITH MATHEMATICS*****

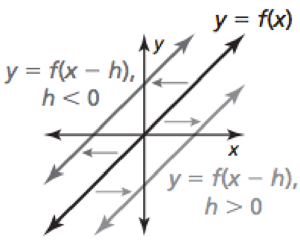
7. The table shows the height *y* of a dirt bike *x* after jumping off a ramp. What type of function can you use to model the data? Estimate the height after 1.75 seconds. What type of function is this? How can you tell? What will the height of the dirt bike be after 1.75 seconds?

Time (seconds), *x*	Height (feet), *y*
0	8
0.5	20
1	24
1.5	20
2	8

1.2: Transformations of Linear and Absolute Value Functions (pg. 12-15)

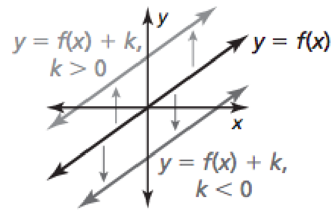
Horizontal Translations

The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.



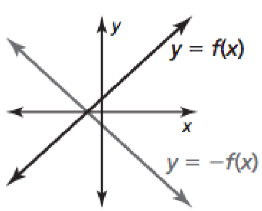
*****CONCEPT 1: WRITING TRANSLATIONS OF FUNCTIONS*****

1. Let $f(x) = 2x + 1$
   a. Write a function *g* whose graph is a translation 3 units down of the graph of *f.*
   b. Write a function *h* whose graph is a translation 2 units to the left of the graph of *f.*

2. $f(x) = 4x - 9$
   a. Write a function *g* whose graph is a translation 5 units up of the graph of *f.*
   b. Write a function *h* whose graph is a translation 1 units to the right of the graph of *f.*

Reflections in the *x*-axis

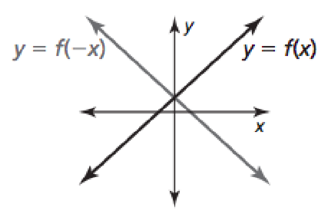
The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.



Multiplying the **outputs** by −1 changes their signs.

Reflections in the *y*-axis

The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.



Multiplying the **inputs** by −1 changes their signs.

*****CONCEPT 2: WRITING REFLECTIONS OF FUNCTIONS*****

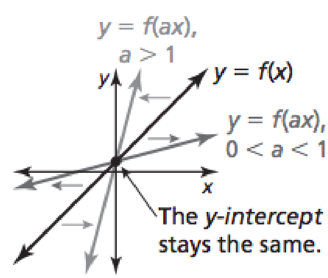
3. Let $f(x) = |x+3| + 1$
   a. Write a function *g* whose graph is a reflection in the x-axis of the graph of *f.*
   b. Write a function *h* whose graph is a reflection in the y-axis of the graph of *f.*

4. Let $f(x) = |x-5| - 4$
   a. Write a function *g* whose graph is a reflection in the x-axis of the graph of *f.*
   b. Write a function *h* whose graph is a reflection in the y-axis of the graph of *f.*

Horizontal Stretches and Shrinks

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

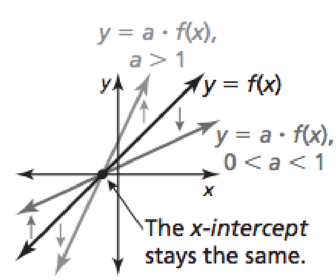
Multiplying the **inputs** by a before evaluating the function stretches the graph horizontally (away from the *y*-axis) when $0 < a < 1$, and shrinks the graph horizontally (toward the *y*-axis) when $a > 1$.



Vertical Stretches and Shrinks

The graph of $y = a \bullet f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

Multiplying the **outputs** by a stretches the graph vertically (away from the *x*-axis) when $a > 1$, and shrinks the graph vertically (toward the *x*-axis) when $0 < a < 1$.



*****CONCEPT 3: WRITING STRETCHES & SHRINKS OF FUNCTIONS*****

5. Let $f(x) = |x - 3| - 5$.
Write (a) a function *g* whose graph is a horizontal shrink of the graph of *f* by a factor of $\frac{1}{3}$, and (b) a function *h* whose graph is a vertical stretch of the graph of *f* by a factor of 2.

6. Let $f(x) = |x - 1| + 2$.
   a. Write a function *g* whose graph is a horizontal stretch of the graph of *f* by a factor of 5.
   b. Write a function *h* whose graph is a vertical shrink of the graph of *f* by a factor of 0.25.

*****CONCEPT 4: COMBINING TRANSFORMATIONS*****

7. Let the graph of *g* be a vertical shrink by a factor of 0.25 followed by a translation 3 units up of the graph of $f(x) = x$. Write a rule for *g.*

8. Let the graph of *h* be a horizontal stretch a factor of 8 followed by a translation 10 units down of the graph of $f(x) = x$. Write a rule for *h.*

*****CONCEPT 5: WORD PROBLEMS*****

8. You design a computer game. Your revenue for *x* downloads is given by $f(x) = 2x$. Your profit is $50 less than 90% of the revenue for *x* downloads. Describe how to transform the graph of *f* to model the profit. What is your profit for 100 downloads?

9. You make cards to sell. Your revenue is represented by $f(x) = 3x$. Your profit is $15 more than 60% of the revenue for *x* cards. What is your profit for 200 cards?

1.3: Modeling with Linear Functions (pg. 22-24)

Writing an Equation of a Line

Given slope m and y-intercept b Use slope-intercept form:

$$y = mx + b$$

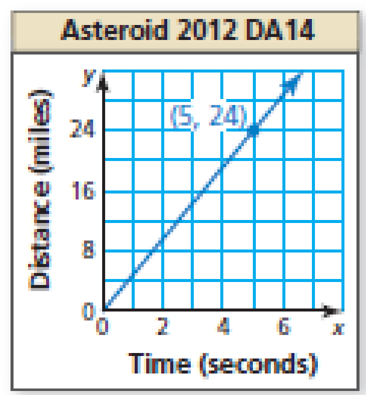
Given slope m and a point (x_1, y_1) Use point-slope form:

$$y - y_1 = m(x - x_1)$$

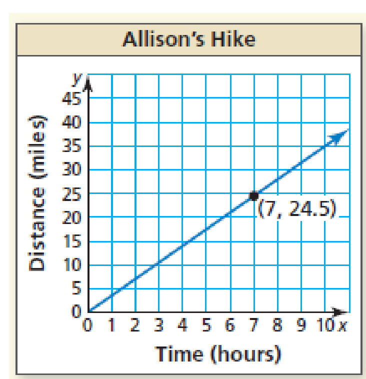
Given points (x_1, y_1) and (x_2, y_2) First use the slope formula to find m. Then use point-slope form with either given point.

***** CONCEPT 1: WRITING A LINEAR EQUATION FROM A GRAPH*****

1. The graph shows the distance Asteroid 2012 DA14 travels in *x* seconds. Write an equation of the line and interpret the slope. The asteroid came within 17,200 miles of Earth in February, 2013. About how long does it take the asteroid to travel that distance?



2. The graph shows the distance Allison hikes in *x* hours. Write an equation of the line and interpret the slope. How long does it take Allison to hike 14 miles travelling at the same rate?



***** CONCEPT 2: MODELING WITH MATHEMATICS*****

3. Two prom venues charge a rental fee plus a fee per student. The table shows the total costs for different numbers of students at Lakeside Inn. The total cost (in dollars) for *x* students at Sunview Resort is represented by the equation $y = 10x + 600$. Which venue initially charges less per student? How many students must attend for the total costs to be the same?

Lakeside Inn	
Number of students, *x*	Total cost, *y*
100	$1500
125	$1800
150	$2100
175	$2400
200	$2700

4. Kelly and Kim are both babysitters. Kelly charges a flat fee of $10 plus $6 per hour to babysit. The table shows the total hourly fee that Kim charges to babysit. Who initially charges more per hour? How many hours must Kim and Kelly babysit for their total fees to be the same?

Kim's Babysitter Fee	
Number of hours, *x*	Total fee, *y*
1	$22
2	$26
3	$30
4	$34

Finding a Line of Fit

Step 1 Create a scatter plot of the data.

Step 2 Sketch the line that most closely appears to follow the trend given by the data points. There should be about as many points above the line as below it.

Step 3 Choose two points on the line and estimate the coordinates of each point. These points do not have to be original data points.

Step 4 Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

***** CONCEPT 3: FINDING A LINE OF FIT*****

5. The table shows the femur lengths (in centimeters) and heights (in centimeters) of several people. Does the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a person whose femur is 35 centimeters.

Femur length, x	Height, y
40	170
45	183
32	151
50	195
37	162
41	174
30	141
34	151
47	185
45	182

6. The table shows the amount of fruit used to make a smoothie (in ounces) and the total cost (in dollars) of the smoothie. Does the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the total cost of a smoothie that is made using 8 ounces of fruit.

Fruit in a smoothie (ounces), x	Total cost, y
2	\$2.10
1	\$1.50
4	\$3.05
7	\$4.45
10	\$5.95
3	\$1.75
6	\$4.00
9	\$5.35

1.4: Solving Linear Systems (pg. 30-33)
A system is a set of equations that you solve all at once

***** CONCEPT 1: SOLVING A LINEAR SYSTEM WITH ONE SOLUTION*****

1. Solve the system.

$4x + 2y + 3z = 12$ Equation 1
$2x - 3y + 5z = -7$ Equation 2
$6x - y + 4z = -3$ Equation 3

2. Solve the system

$x - y + z = -3$
$2x - y + 5z = 4$
$4x + 2y - z = 2$

***** CONCEPT 2: SOLVING A LINEAR SYSTEM WITH <u>NO SOLUTION</u>*****

3. Solve the system.

$$\begin{aligned} x + y + z &= 2 \\ 5x + 5y + 5z &= 3 \\ 4x + y - 3z &= -6 \end{aligned}$$

4. Solve the system.

$$\begin{aligned} x + y + z &= 1 \\ 6x + 9y - 12z &= 14 \\ 12x + 18y - 24z &= -11 \end{aligned}$$

***** CONCEPT 3: SOLVING A LINEAR SYSTEM WITH <u>INFINITE SOLUTIONS</u>*****

5. Solve the system.

$$\begin{aligned} x - y + z &= -3 \\ x - y - z &= -3 \\ 5x - 5y + z &= -15 \end{aligned}$$

6. Solve the system.

$$\begin{aligned} 2x - 2y + 2z &= 12 \\ 10x - 2y + 10z &= 60 \\ 2x + 2y + 2z &= 12 \end{aligned}$$